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Hon. Bette Stephenson, M.D., Minister
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1980

MATHEMATICS

CURRICULUM GUIDELINE FOR THE INTERMEDIATE DIVISION

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Intermediate Division Mathematics Guideline Committee 1976-77

(The positions given are those held by the committee members in 1976-77.)

Bob Alexander, Lecturer, Faculty of Education, University of Toronto; Assistant Head of Mathematics, Lawrence Park Collegiate Institute, Board of Education for the City of Toronto

Paul Atkinson, Head of Mathematics, Elmira District Secondary School, Waterloo County Board of Education

Tom Griffiths, Teacher, A.B. Lucas Secondary School, Board of Education for the City of London

Bob LeRoy, Vice-Principal and Teacher, Polson Park Public School, Frontenac County Board of Education

Jim MacLean, Education Officer, Research and Evaluation Branch, Ministry of Education

Michael McKenna, Education Officer, Niagara Region, Ministry of Education

Paul Pogue, Consultant and Teacher, Barrie North Collegiate Institute, Simcoe County Board of Education

George Scroggie, Co-ordinator, Intermediate Division Mathematics Guideline Project; Education Officer, Curriculum Branch, Ministry of Education

Rosaline Seguin, Assistant Professor of Mathematics, Faculty of Education, University of Ottawa

Michael Silbert, Supervisor of Mathematics, Board of Education for the City of Hamilton

Lorna Strobel, Vice-Principal, Tabor Park Vocational School, Board of Education for the Borough of Scarborough

Peter Weygang, Head of Mathematics, Levack District High School, Sudbury Board of Education

Claire Zeller, Head of Mathematics, Westwood Secondary School, Peel Board of Education

Writing Team 1976-77

The writing team was composed of the following members of the Intermediate Mathematics Guideline Committee:

Full-time: Jim MacLean, Paul Pogue, George Scroggie (co-ordinator), Peter Weygang, Claire Zeller

Part-time: Michael McKenna, Michael Silbert, Lorna Strobel

Validation Committee 1978-79

(The positions given are those held by the committee members in 1978-79.)

Wyn Bates, Director of Mathematics, Board of Education for the City of Toronto

Fred Burford, Principal, Downsview Secondary School, Board of Education for the City of North York

Bill Collier, Principal, St. Florence Catholic School, Metropolitan Separate School Board

Wilma Dowling, Head of Mathematics, Waterdown District High School, Wentworth County Board of Education

Louise Fleming, Education Officer, Curriculum Branch, Ministry of Education

Elaine Harvey, Teacher, Sydenham High School, Frontenac County Board of Education

Jacques Lamoureux, Teacher, MacDonald-Cartier Secondary School, Sudbury Board of Education

Jack LeSage, Teacher, Eastview Secondary School, Simcoe County Board of Education

Michael McKenna, Chairman, Validation Committee; Education Officer, Northeastern Region, Ministry of Education

Marilyn McPhee, Teacher, Ryerson Senior Public School, Board of Education for the City of Hamilton

Alex Norrie, Co-ordinator of Mathematics, Peel Board of Education

Maurice Poirier, Teacher, École Secondaire Garneau, Carleton Board of Education

Sam Pravica, Assistant Professor, Mathematics Department, Faculty of Education, University of Western Ontario

Don Roberts, Head of Mathematics, Beamsville District Secondary School, Lincoln County Board of Education

George Scroggie, Education Officer, Curriculum Branch, Ministry of Education

Peter Weygang, Principal, Thomas Aquinas School, Kenora District Roman Catholic Separate School Board

Claire Zeller, Head of Mathematics, Morningstar Drive Secondary School, Peel Board of Education

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Preface

This guideline sets out the policy for the development of the following courses for Grades 7 through 10. It may be considered optional in 1980-81, but becomes the official guideline in 1981-82.

Grade 7 Mathematics
Grade 8 Mathematics
Grade 9 Advanced Mathematics
Grade 9 General Mathematics
Grade 9 Basic Mathematics
Grade 9 Modified Mathematics, Level 1
Grade 9 Modified Mathematics, Level 2
Grade 10 Advanced Mathematics
Grade 10 General Mathematics
Grade 10 Basic Mathematics
Grade 10 Modified Mathematics, Level 1
Grade 10 Modified Mathematics, Level 2

Effective September 1981, *Mathematics, Intermediate Division, 1980* replaces the following documents:

Mathematics I. 12A, Grade 7	1963 *141-020*
Mathematics I. 12B, Grade 8	1964 *141-030*
Mathematics I. 12C (4 and 5), Grade 9	1965 *141-040*
Mathematics I. 12D (4), Grade 10	1966 *141-050*
Mathematics I. 12D (5), Grade 10	1966 *141-051*
Curriculum R.P. 27 (Mathematics, pp. 250-54), Grades 9 and 10	1963 *140-010*
Curriculum R.P. 31 (Mathematics, pp. 70 and 71), Grade 10	1963 *182-014*
Curriculum R.P. 35 (Mathematics, Years 1 and 2), Grades 9 and 10	1962 *141-090*

Rationale

Introduction

The mathematics program of the Primary and Junior divisions presents an integrated view of mathematics in which concepts of arithmetic, measurement, and geometry are interwoven with practical experiences and uses of mathematics from the child's world. This view is extended in the Intermediate Division, with increased emphasis on consolidating and expanding ideas that were introduced in the earlier years.

The Intermediate Division corresponds to the students' period of adolescence. Because of the physical, emotional, and intellectual changes that occur during this period, adolescents are inclined to think of themselves less as children and more as adults. At the beginning of the Intermediate Division, most of them operate at the semi-concrete stage of development when solving practical problems; by the time they reach Grade 10, many of them will have expanded their ability to deal with abstract thought.

An important feature of the program is the emphasis placed on relating mathematical concepts to the situations in which they are used in everyday life. At each stage of the program, students are encouraged to select or create mathematical models that represent aspects of the real world; to study, analyse, and manipulate these models; and then to relate their findings back to the original situations. In using this process to make sense of their world, students will employ such models as geometric diagrams, numerals and arithmetic operations, algebraic expressions and equations, and graphs and charts that represent data and illustrate relations. These practical experiences should help students to develop confidence in their ability to use mathematical concepts and skills and to appreciate the beauty and power of mathematics.

Aims

The mathematics curriculum for the Intermediate Division is concerned with applying mathematics not only to today's world but also to that of the future. At the same time, it is concerned with the emotional, intellectual, and social development of each student. In keeping with these two central concerns, the mathematics program aims to:

- develop in students the mathematical literacy they will need to lead productive lives now and in the future;
- foster in each student the ability to identify and formulate problems and to solve problems at a level in keeping with his or her stage of mathematical development;
- develop in students an appreciation of the place of mathematics, and its widespread applications, in our culture.

As students mature and feel the need for greater personal autonomy, teachers should provide them with more opportunities for self-direction and self-discipline. The ability to identify and formulate problems and to solve problems in practical contexts is a life skill vital to everyone. Given the nature of adolescence, it is important to provide students at this stage of development with a learning environment that allows and encourages them to develop their thinking patterns and decision-making skills. The learning experiences afforded by such an environment will help them to grow into responsible, independent adults.

Throughout the Intermediate Division, there are many opportunities for students to investigate relevant situations from their world in a search for patterns and for generalizations of their experiences, and to record, discuss, and test their findings for consistency. The arithmetic operations that are essential to the growth of mathematical knowledge and to the application of this knowledge to new situations should be interwoven with and reinforced by concrete activities and applications.

Practice should be given in both oral and written presentations of solutions. Although responsibility for the development of English in the formal sense rests primarily with teachers of English, mathematics teachers share with all other teachers an obligation to maintain good standards of oral and written expression. A major role of the teacher, through patient and sympathetic listening and careful attention to written work, is to search out the student's meaning and to help him or her convey it with clarity and precision.

Some elementary mathematical problems of historical significance, and the approaches taken to them in earlier civilizations, could be discussed with students. Through this knowledge, coupled with modern mathematical applications, students will learn that mathematics is an integral part of our culture and is fundamental to current technological processes.

Program Emphases

This guideline emphasizes that mathematics courses should be of practical value to the students by highlighting concepts and skills that are essential for life today and in the future. This applies to all students at every level of the program.

Courses based upon this guideline should reflect the needs of other subjects (such as science, geography, physical education, family studies, and technical and commercial subjects) and of everyday life. As a base for making informed decisions, students should be able to:

- interpret and use displays of information such as graphs, diagrams, and tables;
- process numerical data efficiently;
- recognize and use algebraic and geometric patterns and properties related to real-life situations.

The curriculum should also help students to understand and apply the skills and mathematical principles that are basic to future studies in mathematics. Generally, the underlying structures of mathematics need not be studied in abstract form in the Intermediate Division; in most instances, these can be experienced in the specific cases in which they occur as the course unfolds.

Experiential Approaches

Experiential approaches are fundamental to the implementation strategies of courses based on this guideline. Because today's students are the citizens of the future, they should learn to examine their world and to understand it without being handicapped by a lack of mathematical knowledge. They should be able to recall and apply mathematical ideas encountered in previous experiences and to use these ideas to solve problems. Thus the mathematics program should include a variety of practical experiences for all grades and for all levels of difficulty. It should make use of concrete materials and investigations, and should emphasize real-life situations that can be simulated in the classroom. In describing the program content in the outlines of topics for each course, the active form is frequently used to indicate that the experiential approach is to be emphasized in the development of this program.

The experiential approach must be complemented by the development of the skills and concepts needed to solve problems that arise in the real world. Conventional terms should be introduced as required, and students should be made aware of the need for a systematic presentation of solutions.

Mathematical Models

Mathematics can be used to simulate many aspects of reality through the creation and use of mathematical models that represent these aspects. While it is usually difficult and expensive to alter real-life objects and situations, models can be easily manipulated. When a model is altered, it often reveals information about the situation that was not known before the simulation activity began. It is for this reason that models are used extensively in science, business, technology, government, and other areas of research, where success often depends upon predictions made from the study of these models.

In helping students to develop model-building skills, teachers should encourage them to investigate simple situations. These situations may relate directly to established models, such as formulae for area, or to less well-defined societal problems. In the Intermediate Division, the emphasis is on the development of such models as graphs, tables, matrices, flow charts, numerical expressions, formulae, equations, geometric drawings, transformations, and statements of probability. Although most of these models are not new to the Intermediate Division, courses based on this guideline should place an even greater emphasis on their development and use.

Physical situations are often represented by models in the form of accurately drawn diagrams. Such models are particularly useful in planning the construction of physical objects. A floor plan of a school, a pattern for a dress, and the architectural plans of a house are examples of geometric models of real objects. The geometry program should use problems in which geometric models, such as the ones suggested here, are essential to the solutions.

Applications

This guideline places a major emphasis on applications of mathematics. For many people this is one of the prime reasons for studying mathematics. Thus the mathematics program for the Intermediate Division should give students many opportunities to see a direct relationship between their studies and the real problems that they face now and in the future.

Everyone uses mathematics, in some form or other, in everyday life. The curriculum outlined in this guideline is intended to help students to recognize these occurrences, to develop an understanding of them, and to extend this understanding to situations that are as yet outside their comprehension.

This requires that students learn the basic skills needed for handling real-life applications. Through this kind of learning, the students' understanding of mathematical principles should improve. Thus students who plan further studies through the Senior Division, and beyond, will have a secure base on which to build more sophisticated ideas and applications.

Applications, and problem-posing and problem-solving, should be interspersed throughout the program rather than developed in isolation. Furthermore, students will appreciate the practical value of the mathematics they study if they are given opportunities to apply their mathematical knowledge in a wide range of subjects. Therefore, an attempt should be made to co-ordinate the mathematics program with programs in other disciplines, such as science and geography, whenever possible.

Although this guideline places a clear-cut emphasis on applications and problem solving, it is the teachers' responsibility to ensure that this emphasis is realized in the classroom. Teachers will need to think in terms of applying mathematics to the environment and to find ways of encouraging their students to view mathematics in the same way.

Calculators are a valuable aid in this regard. They extend the breadth and depth of the applications that can be explored by providing a strong motivational force, by allowing logical thought processes to flow on, and by removing the drudgery of tedious calculations.

This shift towards practical applications applies to courses in all grades and at all levels of difficulty. However, classroom implementation strategies and activities must vary for the different levels in the program and for students of varying mathematical ability. Inappropriate applications can detract from, rather than enhance, the effectiveness of the mathematics program. Thus, whenever possible, applications should be based upon local conditions and activities and be presented at a level of difficulty that is appropriate for the particular groups of students. Curriculum committees could identify applications of mathematics relevant for the local community and then develop classroom materials at various levels of difficulty for use throughout the system.

Virtually every part of Ontario has some local industry, commercial enterprise, or other activity that involves mathematics worthy of exploration. Schools in the north might develop materials associated with mining, forestry, fishing, and natural resources. In the southwest, the furniture industry provides examples of production planning (flow-charting), costing, and geometrical design. In the south, agricultural pursuits provide many relevant applications; for example, cumulative frequency curves related to the growing season, area calculations of field sizes, crop densities, and greenhouse usage could be investigated. In the east, the textile industry and the St. Lawrence Seaway provide applications.

The mathematics of everyday life is primarily concerned with simple business transactions, ranging from the purchase of food and clothing to the negotiation of loans and mortgages. Consumer mathematics incorporates the measurements, estimates, calculations, and decision-making that precede any purchase. Thus, an accurate list of materials, combined with comparative pricing, can effect considerable savings on home improvement projects of all kinds. Accident statistics, inflation, and car insurance rates may be incorporated into a single project that is largely numerical in nature. Such situations give ample opportunity for the use and development of number skills at increasing levels of sophistication as the student moves through the Intermediate Division.

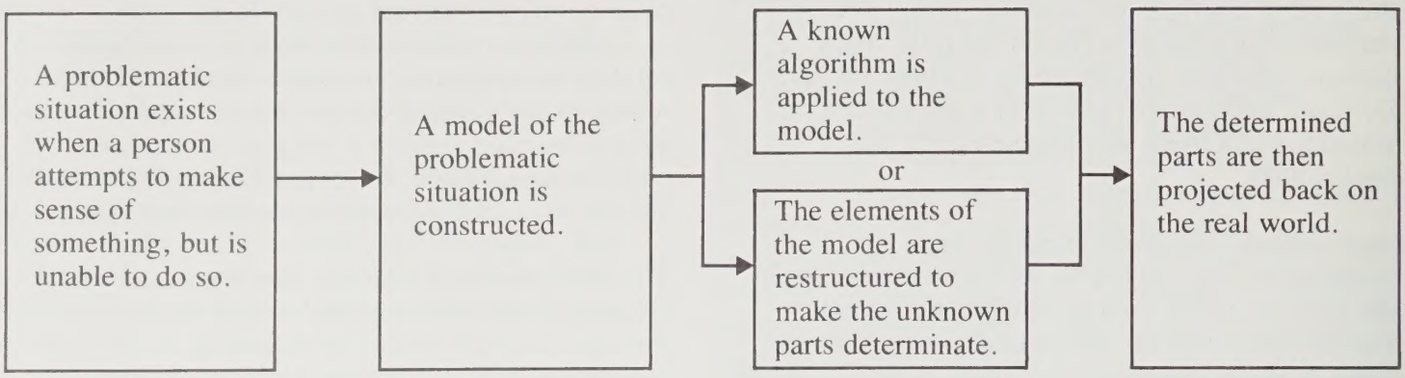
The normal activities around and within the home provide many applications of mathematics which are of lifelong utility. Spreading grass seed or fertilizer involves work with ratios, areas, and concentrations; the indoor gardener faces essentially the same task on a smaller scale when feeding plants. Nearly every recipe, from making porridge to home canning, can be increased or decreased by simple computations. Planning a diet is another activity that involves ratios. (This activity could lead to a consideration of the more exacting problems faced by a hospital dietitian.)

The core part of the Intermediate Division program provides numerous opportunities to use numbers in real-life situations. These applications form unifying links for a variety of topics across the strands and are expected to be an integral part of locally designed programs. These experiences can form the base of a growing awareness of the structures of mathematics.

Problem Solving

Developing the ability to solve problems is the ultimate, but often elusive, goal of education. Problems are solved by drawing on past experiences – sometimes in a systematic manner, but often in flashes of creativity and intuition. Problem solving is not exclusively the domain of mathematics; it is an integral part of all subjects and of everyday life.

Systematic problem solving involves the following stages:*


A problematic situation exists when a person attempts to make sense of something, but is unable to do so.
A model of the problematic situation is constructed.
A known algorithm is applied to the model.
or
The elements of the model are restructured to make the unknown parts determinate.
The determined parts are then projected back on the real world.

Although this progression applies to problem-solving situations in general, it is being considered here in relation to mathematics. In learning to solve problems in a systematic manner, students should:

- develop their ability to identify and formulate problems of a logical-quantitative nature;
- develop their model-building skills;
- develop a widening awareness of the genuine applications of mathematical ideas in both school subjects and real life, as well as an understanding of the uniqueness and limitations of logical thought;
- use increasingly sophisticated models and procedures to deal with problems arising from real-life experiences.

In order to place appropriate stress on problem solving, teachers will need to introduce the above emphases in their mathematics courses. When students are involved in activities of this kind over an extended period of time, they will improve their ability to solve problems in a systematic manner.

* Adapted from a statement in a letter by Floyd Robinson of OISE to the Intermediate Division Mathematics Committee, February 1977.

The Mathematics Program

Introduction

Mathematics, Intermediate Division, 1980 provides the rationale, policy, and mathematical content upon which schools are to base courses specifically designed to meet the needs of their students. The series of charts provided in the guideline illustrates the development of the program at various levels. Chart 1, "The Mathematics Program for the Intermediate Division", gives a capsule view of the entire program and shows the different sequences of courses that comprise it. It illustrates that topics develop sequentially from Grade 7 to Grade 8 and then within each level of difficulty in Grades 9 and 10. Charts 2 to 12 highlight the key mathematical ideas of the courses in the program and provide detailed outlines of the topics for each course.

Levels of Difficulty

Grades 7 and 8

Students entering Grade 7 are at different levels in their understanding of mathematics and in the development of associated skills. They also differ in their attitudes towards the subject. As well, patterns of class organization in Grades 7 and 8 vary from school to school and from school system to school system. Some schools use a complete rotary time-table, while others adopt some modification of it; still others maintain a one teacher–one class pattern of scheduling time. These circumstances require that individual teachers of Grades 7 and 8 be responsible for building courses that take into account the different needs of their students. The objectives and expectations of these courses should be flexible enough to accommodate the students' range of abilities, understanding, and performance levels, however broad this range may be.

Grades 9 and 10

The program for Grades 9 and 10 has been developed at four levels of difficulty in order to meet the full range of student needs in these grades.

The Advanced Program is designed for students who have successfully completed the regular mathematics program in Grade 8 and who show an interest in studying mathematics to the end of Grade 13 and possibly beyond.

The General Program is designed for students who have completed the regular mathematics program in Grade 8, but who have had some difficulty in understanding and consolidating the concepts and skills, and/or who are not planning to pursue studies involving mathematics at the Grade 13 or the university level. Such students may be planning to continue their education in other institutions after completing Grade 12.

The Basic Program is designed for students who have been promoted to Grade 9 on the basis of their overall achievement in Grade 8, but who require additional experiences with the concepts and skills of Grade 7 and Grade 8 Mathematics before taking other mathematics courses at the secondary level. Such students may be planning to enter the work world directly upon leaving high school.

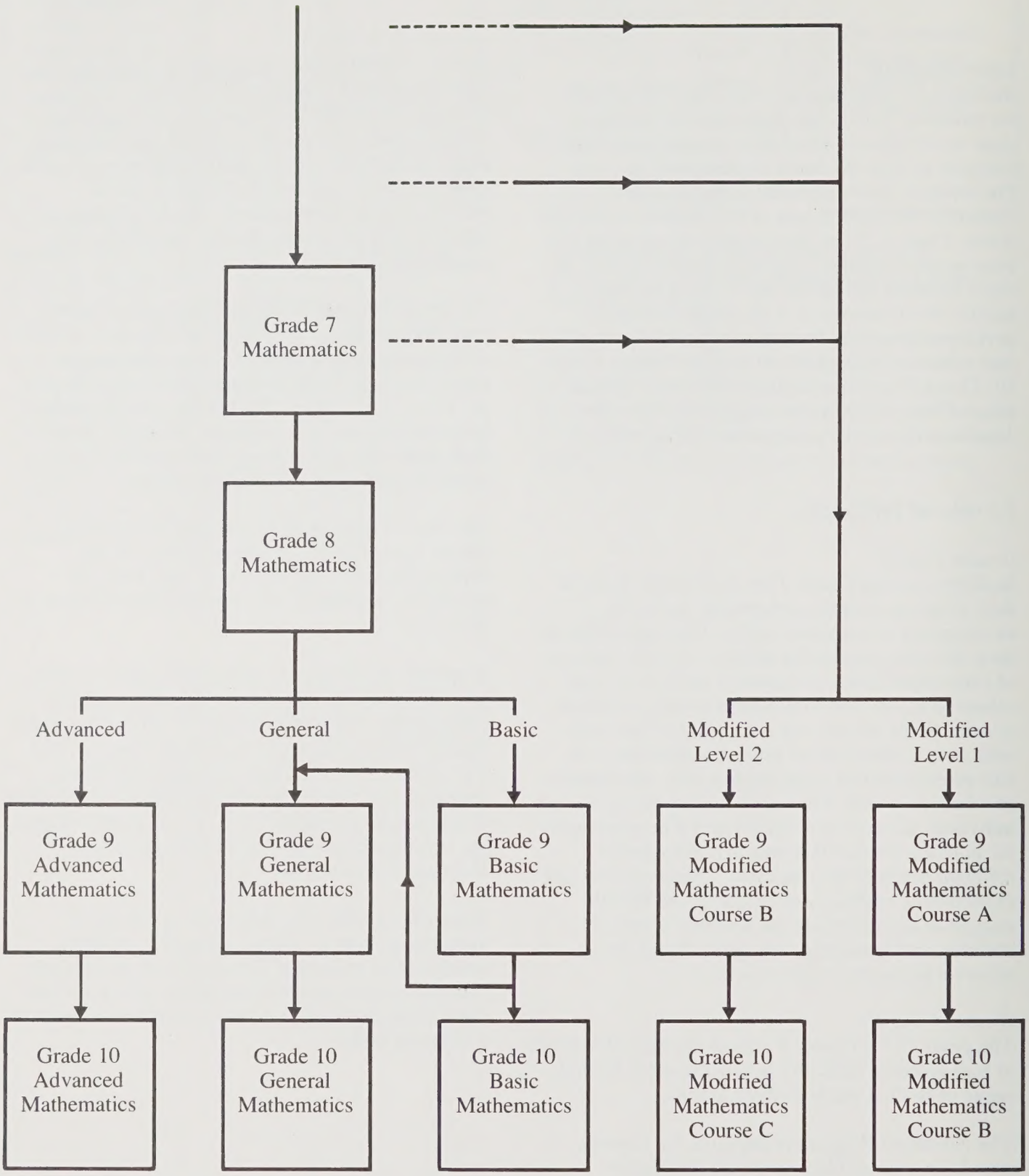
The Modified Program is designed for students whose understanding of mathematics is at the Primary or Junior Division level, and who are enrolled in practical courses at the modified level of difficulty.

A school need not necessarily offer courses at each level of difficulty; local conditions will determine whether this is desirable or feasible. However, in the context of such factors as student enrolment, staff size, and its total academic program, the school should do everything possible to ensure that the range of mathematics courses that it offers accommodates the specific needs, interests, and mathematical abilities of its students.

Generally, in choosing mathematics courses in succeeding years of secondary school, students should select courses at the same level of difficulty. However, there may be cases where it is in the best interests of a student to take his or her next course at a different level.

CHART 1

The Mathematics Program for the Intermediate Division



The above flow chart illustrates the sequences of courses in the Intermediate Division for which the *Outlines of Topics* have been developed. The topics evolve sequentially through Grades 7 and 8 and then within each of the levels of difficulty in Grades 9 and 10.

The Program for Grades 7 and 8

Grade 7 Mathematics

Section	*Numerical Methods*	*Algebra*	*Geometry*
1	Data Graphs	*Communications	Basic Notions in Geometry
2	Number Applications	Equations	*Constructions
3	Fractions, Ratios	Relations	Patterns With Tiles
4	*Percent		*Transformations
5	*Factors		*Enlargements and Reductions
6	Integers		*3-D Geometry
7	Other Arithmetics		
8	*Measurement of Length, Area, and Volume		

Grade 8 Mathematics

Section	*Numerical Methods*	*Algebra*	*Geometry*
1	*Number Applications	*Formulae and Equations	Constructions
2	*Fractions, Ratio, Percent	Relations	*†Transformations
3	Rational Numbers in Decimal Form	Probability	*Properties of Plane Figures
4	*†Data Graphs	Flow Charts	*3-D Geometry
5	*Integers		
6	Measurement of Length, Area, and Volume		
7	*The Circle and Related Measures		

*Some topics in the section are optional.
All topics in the section are optional.
† Full implementation of the section is to begin no later than 1982-83.

The Advanced Program

Grade 9 Advanced Mathematics

Section	*Numerical Methods*	*Algebra*	*Geometry*
1	Number Applications	Flow Charts	*Constructions, Applications
2	*††Statistics	*Polynomials	*††Coordinate Geometry
3	*Powers, Roots	Operations With Rational Expressions	*††Congruence Transformations
4	*Rational Numbers	Equations	Dilatation
5	*††Applications of Percent, Ratio, and Measurement	*Relations	Displacement
6	Lists, Tables	Probability	

Grade 10 Advanced Mathematics

Section	*Numerical Methods*	*Algebra*	*Geometry*
1	Radicals	*Equations and Inequations	*Coordinate Geometry
2	Applied Trigonometry	*Polynomials and Rational Expressions	*†††Coordinates and Transformations
3	*†††Statistics	Relations	*†††Isometries
4	Matrices	Probability	*†††Deductive Geometry
5		Computers	*Vectors
6			*3-D Geometry

*Some topics in the section are optional.
All topics in the section are optional.
†† Full implementation of the section is to begin no later than 1983-84.
††† Full implementation of the section is to begin no later than 1984-85.

The General Program

Grade 9 General Mathematics

Section	*Numerical Methods*	*Algebra*	*Geometry*
1	Number Applications	*Problem Solving and Algebraic Strategies	*The Circle
2	*††Statistics	Polynomials	*Coordinate Geometry
3	*Powers, Roots	Relations	Congruence Transformations
4	*Rational Numbers	Probability	*††Dilatation
5	Percent		Displacement
6	*Applications of Ratio and Measurement		

Grade 10 General Mathematics

Section	*Numerical Methods*	*Algebra*	*Geometry*
1	Number Applications	*Relations	Geometric Figures and Properties
2	Ratio and Rate	*Linear Equations and Inequations	Coordinate Geometry
3	‡‡‡Statistics	*Polynomials	Vectors
4	*Square Roots		
5	Personal Finance		
6	Payrolls and Flow Charts		
7	Applied Trigonometry		
8	Matrices		

*Some topics in the section are optional.
All topics in the section are optional.
††Full implementation of the section is to begin no later than September 1983-84.
†††Full implementation of the section is to begin no later than September 1984-85.

The Basic Program

Grade 9 Basic Mathematics

Section	*Numerical Methods*	*Algebra*	*Geometry*
1	*The Counting Numbers	*Problem Solving	Basic Notions of Geometry
2	Computation With Whole Numbers and Decimals	Introduction to Computing	Constructions
3	Fractions	The Mathematics of Chance	Transformations
4	Ratio and Rate		3-D Geometry
5	Percent		
6	Data Presentation		
7	Powers and Roots		
8	Integers		
9	Measurement		

*Some topics in the section are optional.
All topics in the section are optional.

Grade 10 Basic Mathematics

This course is based on the topics of Grade 9 Basic Mathematics, developed in the context of relevant applications.

It is suggested that these applications be developed from themes such as those outlined in the following resources:

a) *Grade 10 Basic Mathematics, Resource Module 4: Themes and References for Their Development*
b) *Grade 9 Basic Mathematics, Resource Module 2: Themes*
c) *Grade 9 Basic Mathematics, Resource Module 3: Mathematics and the Car*

Other themes may be developed locally.

The themes and/or subthemes used in Grade 10 Basic Mathematics should not duplicate those used in Grade 9 Basic Mathematics.

The Modified Program

Modified Mathematics – Course A
Level 1, Grade 9

Section	*Numerical Methods*	*Geometry*
1	*Non-computational Use of Numbers	Motion Geometry
2	*Computational Use of Whole Numbers	*Geometric Constructions
3	Computational Use of Decimals	Prisms and Pyramids
4	Percent	
5	Aids to Solving Mathematical Problems	
6	*Fractions	
7	*Data Graphs	
8	*Measurement	
9	Probability	

Modified Mathematics – Course B
Level 1, Grade 10
Level 2, Grade 9

Section	*Numerical Methods*	*Geometry*
1	Non-computational Use of Numbers	*Motion Geometry
2	Computational Use of Whole Numbers	*Geometric Constructions
3	Computational Use of Decimals	Prisms and Pyramids
4	*Percent	
5	Aids to Solving Mathematical Problems	
6	Fractions	
7	*Data Graphs	
8	*Measurement	
9	*Probability	

Modified Mathematics – Course C
Level 1, Grade 11
Level 2, Grade 10

Section	*Numerical Methods*	*Geometry*
1	Non-computational Use of Numbers	*Motion Geometry
2	Computational Use of Whole Numbers	*Geometric Constructions
3	Computational Use of Decimals	Prisms and Pyramids
4	*Percent	
5	Problem Solving	
6	Fractions	
7	*Data Graphs	
8	*Measurement	
9	*Probability	

*Some topics in the section are optional.
All topics in the section are optional.

Outlines of Topics

Strands, sections, topics
The topics intended for study in each course are listed in Charts 2-12 given further on in this guideline. These topics are grouped by sections under the appropriate strands – numerical methods, algebra, or geometry.

The order in which the strands, sections, and topics are set out in the various charts is not necessarily intended to indicate the order in which they are to be studied during the year. The topics may be linked in many ways, both within any one strand and among the strands. Because real-life problems do not fall neatly into the separate domains of arithmetic, algebra, geometry, statistics, trigonometry, and so on, the courses should stress the connections among the concepts and skills of the different strands, and use them in a variety of applications and problem-solving situations. These connections should be established when courses are planned and developed so that students may experience a unified view of mathematics.

Core and optional topics
Some topics are core and some are optional. Schools must ensure that each course includes all the core topics designated for that course. In most cases, these courses should also include at least some optional topics.

Core topics
Many of the core topics review and extend ideas introduced in previous grades. Thus they provide continuity with the students' earlier experiences and help students to expand their mathematical knowledge. The core topics also provide the foundation needed for future mathematics courses and for the study of other subjects. Not all the students in a particular course will need the same detailed development of the core topics. Consequently, some flexibility should be provided at the classroom level to ensure that all students have experiences consistent with their particular needs.

Optional topics
Optional topics may be used to enrich the students' understanding of core topics, to preview on an informal level topics that will be introduced in future courses, to extend ideas that have been investigated in earlier courses but that do not form part of the core of the course in progress, and to provide enrichment and motivation through recreational mathematics. In planning a particular course, teachers are not restricted to the optional topics listed in the outlines; they may select topics from other outlines of topics or use optional topics of their own choice.

Program Strands

Numerical Methods

In recent years there has been increased emphasis on numerical methods as a basic strategy for dealing with the problems faced by consumers, industries, businesses, governments, economists, researchers, and so on. This circumstance makes it vital that the curriculum at all grade levels provide students with a variety of experiences that:

- enhance their awareness of the roles played by numbers in their lives;
- extend their knowledge and understanding of numbers while consolidating computational skills and skills for processing numerical data;
- increase their number sense and confidence in their ability to use numerical methods for solving relevant problems.

The numerical methods strands of the various courses are intended to serve these purposes. The depth of study that is feasible and the variety and complexity of the situations that can be investigated will depend on the extent to which calculators are used in the program.

Most of the mathematical topics associated with numerical methods have been a traditional part of the program for Grades 7 to 10 for many years. They involve whole numbers, decimals, fractions, ratio, rate, percent, integers, powers, rational and irrational numbers, roots, radicals, and measurement. The concepts and skills associated with these topics are fundamental to the emphasis to be placed in this program on experiential approaches, modelling, applications, and problem solving, as described earlier in the guideline. The topics in the sections dealing with data graphs, statistics, and matrices are new to the program.

Measurement
The sections on measurement in Grades 7 and 8 blend ideas of number, algebra, and geometry. They are included under numerical methods because they provide many opportunities to refine the numerical concepts and skills associated with reading scales, estimation and rounding, computation with whole numbers and decimals, and numerical investigations. Although the emphasis is on numerical skills, these sections overlap and reinforce algebraic ideas related to formulae as generalizations of numerical patterns and geometric ideas related to one-, two-, and three-dimensional figures, as well as their nets, perimeters, areas, and volumes.

Statistics
It is very important today for the average citizen to be able to recognize, understand, and assess the statistical information that appears in various forms in the media and in advertising. This information is sometimes misleading and must be carefully evaluated before being allowed to influence important personal decisions. Further, statistics are used in many subjects both in the secondary school program and at post-secondary levels. Thus it is important for students to deal intelligently with simple statistical ideas.

Initially, in Grades 7 and 8, the emphasis is placed on various ways of collecting, organizing, and representing data by charts and graphs, and on reading and interpreting the information in these representations. This emphasis is refined in the secondary school and extended to include sampling, measures of central tendency, and inferences about the population based on a study of samples. In presenting these ideas, teachers should avoid formal terminology and should make liberal use of experimental situations and of models that represent real situations. This is especially important in dealing with measures of central tendency.

Matrices
Lists and tables are frequently used in daily life to display data systematically. Sports data such as team standings are presented in tables, which are updated frequently by the addition of new information. In stock-taking, the number of individual items and the value of each item can be summarized in arrays; the total value of the items can then be determined by multiplying the corresponding entries in the two arrays and then adding the products. Travel timetables, conversion tables, and lottery arrays are other examples of data stored in lists or tables. All of these can be used to provide students with experiences with matrices without using formal terminology. The formal study of a matrix as a mathematical entity, and of matrices from a structured viewpoint, is not suggested until the Senior Division.

Since they are simply arrays of numbers, matrices can be stored in a computer and manipulated with ease. For this reason, matrices have become one of the most powerful tools in the field of applied mathematics, as seen in such disciplines as medicine and economics. Matrices provide unity in modern mathematics as they draw together ideas from geometry and algebra and deal with them by numerical methods.

Applied trigonometry
Applied trigonometry related to right-angled triangles is optional in Grade 10 Advanced Mathematics and Grade 10 General Mathematics. This work is useful in the study of some technical subjects and of physics. It shows how geometric problems may be solved using numerical methods.

Calculators
The increase in the use of numerical methods, in conjunction with electronic machines, to solve problems and to process data is rapidly affecting the personal lives of many people. The low cost of calculators today and the dropping costs of programmable calculators and micro computers have brought these machines within the reach of many people. Almost everyone owns or has access to a calculator of some type, yet few people know how to use these machines to their full potential.

It is inevitable in a curriculum that deals with realistic situations that students will face problems involving complicated calculations. The use of calculators in these cases removes much of the drudgery of arithmetic and allows more time for the planning of solutions and the logical thinking that is an integral part of mathematics.

Calculators may also be used to introduce, reinforce, or illuminate new concepts. They can help students to:

- complete discovery exercises;
- check their oral or written exercises;
- investigate number patterns;
- apply estimation and rounding skills;
- process extensive numerical data;
- evaluate algebraic expressions;
- substitute numerical values in formulae;
- solve equations;
- make numerical investigations of geometric situations;
- generate and solve problems;
- carry out algorithmic processes, and generate and understand these processes;
- determine values of functions and inverse functions.

Students should be encouraged to develop algorithms to be used with calculators. These can be represented by diagrams or flow charts. Such activities could be used as an introduction to computer programming.

Teachers should explore and evaluate ways of using calculators for the above purposes and for investigating many other topics in the curriculum, such as terminating and non-terminating decimals, operations with integers, powers with integral exponents, compound interest, approximation errors resulting from truncation and from rounding, addition

of fractions, percentage, and so on. When used in such contexts, calculators can provide students with motivation and new insights into the curriculum. The long-range effects of the use of calculators on students, however, will only be determined after years of classroom experience.

Care must be taken to ensure that students do not rely exclusively on calculators for arithmetical computation. A variety of experiences requiring mental arithmetic and simple written calculations should still be included in the mathematics program. In fact, part of the classroom strategy should be to structure situations in which the alert mind can outperform the machine. There is no question that in general the human brain can outperform the machine in doing simple calculations. The point at which the calculator should take over will have to be determined on an individual basis.

Algebra

Algebra has traditionally occupied a central position in the secondary school mathematics curriculum. Its fundamental concepts and skills are essential to the development of many aspects of secondary and post-secondary mathematics, and have many applications in other subjects and in the real world.

Today, computers and other electronic machines can be programmed to provide almost instantaneous solutions to complex problems of a numerical, algebraic, or geometric nature. From a practical viewpoint, some of the routines of traditional algebra, although significant to the structure of the subject, now have limited applications in solving real-life problems. The algebra curriculum, both in its content and in the mathematical approaches taken to it, is designed to reflect these considerations, while preserving the fundamental ideas that are essential to its long-range development.

Algebraic models
Extensive use is made of algebraic models in many applications of mathematics. The development of these models requires a thorough grasp of the concept of variable and of operations with variables, as well as the ability to convert algebraic expressions into forms that are compatible with the capabilities of computers and other electronic machines. Thus it is important to introduce students to algebraic expressions and equations as generalizations of numerical patterns, and to the variables in these expressions as representations of numbers that have real-life significance. Since the coefficients and values of the variables are seldom simple numbers in real-life contexts, it is important that students develop efficient techniques for evaluating expressions and for solving equations that are applicable in both simple and complex cases.

Aspects of the algebra program that lend themselves to building and using algebraic models include: communications (Grade 7); variables; converting word statements to mathematical notation; mappings and arrow diagrams; flow charts; algorithms; formulae generalized from numeric cases; substitution in formulae and algebraic expressions; manipulating formulae; polynomials in nested form; business formulae; formulae for perimeter, area, and volume (numerical methods); solving equations by systematic trial; solving word problems; solving systems of equations using matrices; graphing equations and inequations; graphing systems of equations and inequations; transformations as mappings (geometry) and as matrices (numerical methods); applications in real-life situations; and problem solving.

Substitution
The evaluation of polynomials and other algebraic expressions for particular values of the variable(s) is introduced in Grade 8, along with substitution in formulae. These ideas are extended in Grade 9 in the advanced and general courses.

Manipulative skills
The manipulative skills involved in operations with polynomials and rational expressions and in factoring are introduced gradually, beginning in Grade 9 Advanced and General Mathematics and continuing in Grade 10. They will be further developed in the Senior Division. Emphasis is placed on equivalent algebraic expressions – different expressions that generate the same set of numbers for the same replacement set. The operations with polynomials and with rational expressions, as introduced in Grades 9 and 10, produce equivalent expressions in simplified form. For example, the expanded form $x^2 + x - 6$, factored form $(x + 3)(x - 2)$, and nested form $(x + 1)x - 6$ of an expression are equivalent. The nested form simplifies the substitution process, particularly when the coefficients and substitution values are intricate. The substitution involves the fewest number of steps and can be performed directly on a four-function calculator without memory.

While it is important for students to have experiences in manipulating algebraic expressions, it is advisable in Grades 9 and 10 to use simple examples and exercises. This will help students to understand the principles involved and to be successful in applying them. Once the principles have been fully grasped and the students have developed confidence in their ability to apply them, the skills needed for manipulating expressions can be further developed in the Senior Division.

Solving equations

It is important for students to understand the meaning of the solution of an equation and of a system of equations, and to develop a variety of techniques for finding the solution. These techniques have significant applications in solving problems in all branches of mathematics, in other subjects, and in real-life situations. The development of the understanding and skills necessary for solving equations, and of an awareness of the practical applications of equations, extends across the Intermediate Division into the Senior Division.

In Grade 7, notions of variable, the conversion of word statements to mathematical form, and the solution of equations in one variable by systematic trial are introduced. This is extended in Grade 8 to include the development of formulae by generalizing from numeric cases, substitution techniques, solution of equations by a variety of methods, and the solution of inequations by inspection.

In Grade 9, algebraic methods of solving equations and inequations are introduced along with word problems, and additional emphasis is given to formulae as models of real-life situations. In Grade 10, graphs of linear equations and inequations are drawn. Systems are solved both graphically and algebraically and are applied in practical situations.

Systematic trial

The systematic trial method for solving equations is introduced in Grade 7 and extends through to Grade 9. This method, when coupled with the nested form of the polynomial, has universal application to polynomial equations of any degree. Although special algebraic methods apply to first- and second-degree equations, these methods become complicated for second-degree equations when the coefficients and/or values of the variable are intricate to work with, and they do not work for higher-order equations. Systematic trial must then be used.

Systematic trial reinforces the fundamental concept of a "solution of the equation". Either the selected value of the variable satisifies the equation, or it does not. If it does not, then the tested value is systematically refined until, for all practical purposes, a solution is found. This method is particularly useful when used in conjunction with a calculator or computer.

Flow charts

Flow charts provide an effective way of displaying step-by-step procedures in a variety of situations, both mathematical and non-mathematical. Essentially, they are visual models of the procedures. As such, they give a sense of security to students who have difficulty with sequential patterns, whether the patterns are associated with algorithms, logical reasoning, or problem solving, or are of a general organizational nature. Flow charts are used extensively outside the realm of mathematics to display sequential events from the past, organizational plans, and time-lines. Some examples of these uses might be included in the mathematics program.

Topics related to flow charts are included in all courses at all levels of difficulty. They are part of the core program in Grade 9 General Mathematics and are optional in the other courses. Generally, students who are experiencing difficulty with sequential or organizational procedures will benefit most from the semi-concrete support of flow charts. Teachers should determine the extent to which their students are likely to benefit from the inclusion of these optional topics in the program.

It is expected that an informal approach will be used in presenting this topic. Rectangular boxes connected by arrows may be employed to show sequential steps. The advisability of introducing conventional flow chart boxes, such as start, stop, and decision boxes, is left to the discretion of the teacher. In Grades 8 and 9, inverse flow charts are suggested as an introduction to solving first-degree equations in one variable. This method is effective for many students and reinforces notions of inverse operations.

Probability

Probability deserves consideration in a curriculum that emphasizes that the topics studied be relevant to the students' experiences. Simple probability statements are made every day – in conversation, in newspapers and magazines, in advertising, and on radio and television. The likelihood that an event will occur is a consideration fundamental to most day-to-day decisions, whether such decisions are of a personal nature or of great social, economic, political, or technical significance. Even a simple introduction to probability will help students assess the quantitative aspects of the statements of probability they hear every day.

The basic ideas of probability (which are optional in most of the courses in the program) may be developed through an informal approach involving simple experiments. These principles can then be used to analyse critically statements of probability drawn from the media and other everyday sources. In order to show students that probability is essentially a modelling procedure, the experiments themselves can be discussed as simulations of real-world situations. The data obtained from the

experiments are statistical in nature and can be represented in a variety of tabular and graphical forms. These data can then be used to make statements of probability on which further inferences and predictions may be based.

Geometry

Geometry fulfils a significant role in the mathematics curriculum as it adds an important dimension to our mathematical perception of the world. Numerical methods and algebra enable us to deal with the quantitative features of our environment; geometry allows us to deal with its physical features. Modern trends in the development of geometry as an applied science reach beyond the traditional constraints of geometry as a logical-deductive system. They embrace the numerical and algebraic techniques associated with measurement, transformations, vectors, co-ordinates, trigonometry, graphs, matrices, and calculus. The techniques of building mathematical models to represent aspects of the physical world now blend geometric figures and their properties with numerical and algebraic representations and analysis.

Content
The geometry program of the Intermediate Division provides a gradual consolidation of the students' knowledge of geometric techniques and properties. As such, it acts as a transition between the informal experiences suggested by *The Formative Years* and the refinement, extension, and application of the students' mathematical knowledge in the Senior Division. The program for Grades 7 to 10 evolves from experiential approaches and inductive reasoning in a variety of contexts towards deductive reasoning in Grade 10 Advanced Mathematics. Students should be given many opportunities to investigate geometric properties in specific, tangible contexts and then to make generalizations from these investigations through group and class discussion. These generalizations should be related back to the real world, whenever possible.

These experiences will give students the opportunity to practise the skills of geometric modelling. Such models include:

- diagrams to represent physical features of the real world;
- transformations to describe "before" and "after" positions of physical motions, of enlargements and reductions, and of distortions;
- nets and skeletons of three-dimensional objects;
- diagrams on a coordinate grid with numeric properties such as slope and distance;
- vectors (arrows or ordered pairs) that describe displacements and force;
- mapping rules to describe transformations;
- linear equations and their graphs to describe direct variation and partial variation situations.

Many of the topics in the revised geometry program have been included in previous Intermediate Division courses. These include constructions; classification of angles, triangles, and quadrilaterals; polygons; properties of plane figures; coordinate geometry; conditions for congruent triangles; perpendicular and parallel lines; deductive geometry; the Pythagorean Theorem; and some aspects of transformations, vectors, and three-dimensional figures. The program is now extended to include a more detailed discussion of transformations and symmetry and their applications in the development of many of the other topics in the program; patterns with tiles; scale drawings; physical models of three-dimensional objects; the techniques of sketching three-dimensional figures; a variety of construction techniques; coordinates and transformations; composites of two and three reflections; and distortions. Many of these topics are optional parts of the program, and their general development is intended to be informal.

Constructions
Topics related to constructions are introduced in Grade 7. They extend across the division and are both core and optional parts of the program. Since the program emphasizes an inductive approach to properties, it is important that students be able to construct plane figures with reasonable accuracy. A variety of methods using different materials is suggested. In this way, the properties of the constructed figures will be consolidated, and students will be able to choose for themselves the most efficient construction methods. These construction techniques are used when investigating transformation images, both to construct the images and to determine their properties.

Transformations
The topics related to translation, reflection, rotation, line symmetry, and rotational symmetry extend the investigations of slides, flips, turns, and symmetries of the Junior Division. These transformations are introduced in Grade 7 as mappings that establish correspondences between the "before" and "after" representations of physical motion. In fact, they are mathematical models of these motions.

Techniques for constructing the images, tests for determining the type of congruence transformations, and an understanding of the properties of the transformations are also developed in Grades 8 to 10. The study of transformations and symmetry can

be used as the basis for classifying triangles and quadrilaterals, establishing properties of figures, and constructing figures, and as part of the strategy for proving deductions.

Half-turn is introduced in Grade 9 Advanced Mathematics and Grade 9 General Mathematics. Glide-reflection is introduced and the meaning of isometries established in Grade 10 Advanced Mathematics. Composites of two and three reflections are also introduced at this level as an option.

Enlargements and reductions, and distortions as an option, are introduced in Grade 7, using semi-concrete materials; dilatation images are presented in Grades 8 and 9. All of these illustrate transformations that do not preserve congruence. (Scale drawings and other applications are used to illustrate the everyday occurrences of these ideas.)

Coordinate geometry
Notions of coordinates are introduced informally in the Junior Division and in Grades 7 and 8, where activities with geoboards, dot paper, and various grids are used to determine properties of motions, transformations, symmetries, and plane figures. Line graphs in the first quadrant are studied in Grade 8 statistics.

Basic ideas of coordinate geometry, on a four-quadrant grid, are developed in Grades 9 and 10. These are extended to the investigation of transformation images on a grid and the mapping rules for these transformations. The properties of plane figures may be investigated using a coordinate grid in Grade 10 Advanced Mathematics. Matrices for special cases of transformations are suggested as an optional topic at this level. This work is continued in the Senior Division, where these techniques may be used for an efficient study of functions and relations and their graphs and properties.

Vectors
Displacements are suggested for optional study in Grade 9. Vectors, defined by arrows and by ordered pairs, are introduced in Grade 10 and applied to force and navigation problems. Vectors may also be used as an optional topic to investigate the properties of geometric figures.

The introduction of vectors will prepare students for the applications of vectors in Grade 11 physics and will illustrate once more the use of mathematical models in dealing with real-life situations.

Deductive geometry
Deductive geometry is introduced in Grade 10 Advanced Mathematics. Here the students develop patterns of sequential reasoning based on geometric properties that have been established informally in earlier courses. The base for this work is a blend of properties of plane figures, transformations and symmetries, coordinate geometry, and possibly vectors and/or matrices. The choice of strategy for developing patterns of deductive reasoning is left to the discretion of individual teachers or schools. The approach may be that of traditional Euclidean geometry; it may be modified to evolve from the properties of transformations and symmetries; it may incorporate other approaches; or it may be a composite approach. Whatever approach is chosen, students should be given opportunities to use the full range of knowledge acquired in their earlier studies. It is not expected that this work will be rigorous or stress formal axiomatic systems; such a treatment is beyond the intended scope of the Grade 10 Advanced Program for most students.

Three-dimensional geometry
Through activities and investigations, students are given opportunities to examine real-life objects and to relate them to geometric three-dimensional figures; to construct models of these objects; to examine their properties; and to draw and interpret two-dimensional sketches of them. In this way, students are encouraged to think about objects in their environment and to represent them geometrically, and thus to sharpen their spatial perceptions.

The Program for Grades 7 and 8

Locally developed courses

The mathematics program for Grades 7 and 8 is to consist of courses that are developed locally from the outlines of topics in Charts 2 and 3 (see pages 18 and 19) and that are consistent with the policy and directions set out in this guideline. These courses are to provide a consolidation, refinement, and extension of the ideas developed in the Primary and Junior programs, based on *The Formative Years*, and are to be sufficiently flexible to meet the needs of the range of students within each class.

Implementation

Programs based on *The Formative Years* have only been in effect since 1975, and it will be at least 1981 before students entering Grade 7 will have had their entire school experience based on the ideas contained in that document. As a result, the contents of the current Grade 7 course overlap the program outlined in *The Formative Years*. In the first few years of the new program, most of the time should be devoted to developing the core topics as a consolidation and extension of the Primary and Junior Division programs. As the policies of *The Formative Years* become more fully implemented, the program for Grades 7 and 8 should be modified to incorporate more of the optional topics.

The program for Grade 8 may be implemented in two stages. Most of the core topics are to be introduced in 1981-82, but the sections on data graphs and transformations need not be implemented until September 1982. The reason for this is that the topics in these sections are new to the Intermediate Division and are developed sequentially within the revised program. Thus they cannot be effectively implemented as suggested until incoming students have the necessary background from Grade 7. The delayed implementation of these sections, however, should not cause a vacuum in the program. During 1981-82, Grade 8 teachers are encouraged to introduce some aspects of data graphs and transformations by selecting some of the topics from the Grade 8 course and teaching them in conjunction with their prerequisite topics from the Grade 7 course.

Once the mathematics background of students entering Grade 7 is fully based on *The Formative Years*, it is expected, under normal circumstances, that 75 percent of the available course time will be required for the development of the core topics. In the accompanying chart, the green numerals beside the sections that include core topics represent a suggested number of hours of instructional time for these core topics.

The Program for Grades 7 and 8

Grade 7 Mathematics

Section	*Numerical Methods*	*Algebra*	*Geometry*
1	Data Graphs 5	*Communications 3	Basic Notions in Geometry 6
2	Number Applications 21	Equations 5	*Constructions 2
3	Fractions, Ratios 9	Relations	Patterns With Tiles
4	*Percent 15		*Transformations 5
5	*Factors 4		*Enlargements and Reductions 4
6	Integers		*3-D Geometry 4
7	Other Arithmetics		
8	*Measurement of Length, Area, and Volume 10		

Grade 8 Mathematics

Section	*Numerical Methods*	*Algebra*	*Geometry*
1	*Number Applications 12	*Formulae and Equations 11	Constructions 12
2	*Fractions, Ratio, Percent 18	Relations	*†Transformations 7
3	Rational Numbers in Decimal Form	Probability	*Properties of Plane Figures 5
4	*†Data Graphs 8	Flow Charts	*3-D Geometry 4
5	*Integers 7		
6	Measurement of Length, Area, and Volume 10		
7	*The Circle and Related Measures 4		

*Some topics in the section are optional.
All topics in the section are optional.
† Full implementation of the section is to begin no later than 1982-83.

Emphases
Because it is important that all students receive a variety of experiences designed to consolidate and extend their knowledge of numbers, their number sense, and their ability to use numbers in a variety of applications and problem-solving situations, the numerical methods component of each course is intended to permeate the course, both in the planning stage and in its implementation. Many of the sections in the algebra and geometry strands contain opportunities for developing number applications. A judicious use of games, puzzles, and other activities can do much to motivate students while subtly extending their number sense and consolidating their skills. A significant change in the program occurs in the geometry strand, where the geometry topics of *The Formative Years* are extended and a major emphasis is placed on experiential approaches.

Levels of difficulty
It is recognized that students entering Grade 7 are at different levels of mathematical knowledge and understanding. This diversity requires that courses be planned at different levels of difficulty for Grades 7 and 8. While each of these courses will be based on the same core topics, the topics themselves may be developed in different ways and with different expectations. The breadth and depth of treatment, approaches, methodologies, examples and applications, evaluation techniques, and situations investigated should all be tailored to the specific needs of the students concerned. The courses may also incorporate different optional topics. The initial planning of courses at different levels of difficulty can be done for the entire school system by a local curriculum committee or by groups of teachers in individual schools. The refinement of this general plan rests with individual teachers whose task it is to adapt the courses to the specific needs of their students.

Special education
Teachers planning special education courses need not follow the core-option program as outlined in this guideline. They may build their courses from topics taken from *The Formative Years*, the Grade 7 and 8 program, and other sources that are helpful in designing courses for students with special needs.

Continuity in planning
Teachers should ascertain the levels of development of their incoming students so that classroom experiences may be planned effectively. Records should be kept of the content and scope of any courses that have been significantly modified and of the achievement of individual students, both to assist subsequent teachers in building their courses and to enable them to offer students maximum assistance.

Because growth in mathematical understanding should be continuous and coherent, it is vital that teachers of different grades share the responsibility for developing courses. This is relatively easy to organize in K-8 schools where students from the Primary, Junior, and Intermediate divisions are housed in the same building. When teachers in these divisions are situated in two or more locations, more time and effort, and hence a greater commitment, are required to maintain a continuing dialogue. In any event, joint planning is essential if the goals of the program are to be achieved.

Grade 7 Mathematics

Numerical Methods (N)

Section 1. Data Graphs
a) Collecting data, tally charts
b) Representing whole number data by pictographs and bar graphs
c) Reading and interpreting pictographs and bar graphs

Section 2. Number Applications
a) Place value
b) Operations with whole numbers and decimals; reasonableness of results; estimating answers; applications in real-life situations
c) Rounding; applications in real-life situations
d) Multiplying and dividing a number by a power of 10
e) Using properties of arithmetic operations such as the commutative, associative, and distributive properties, and the roles of 0 and 1 to simplify numerical expressions

Section 3. Fractions, Ratios, Rate
a) Concept of a fraction
b) Operations with fractions, reciprocals; applications to real-life problems
c) Ratios as comparisons
d) Equivalent ratios; applications in practical problems
e) Rate, developed from real-world examples

Section 4. Percent
a) Percents as hundredths
b) Conversion of percents to decimals, and decimals to percents
c) Applications to sales tax, simple interest, discount
d) Applications to gain or loss

Section 5. Factors
a) Composite numbers in factored forms, and in prime factored form
b) Powers; evaluation of powers
c) Divisibility tests for numbers such as 2, 3, 4, 5, and 9
d) Lowest common multiple

Section 6. Integers
a) Establishing the need for integers from practical examples
b) Locating integers on the number line; order
c) Adding and subtracting with integers

Section 7. Other Arithmetics
a) Modular arithmetics in real life: 12-and 24-hour clocks; week; month; year; 360° in a complete rotation
b) Developing numeration bases using activities with concrete materials
c) Adding and subtracting using bases less than 10

Section 8. Measurement of Length, Area, Volume
a) Estimating and measuring lengths of line segments and curves, and the perimeters of figures
b) Perimeter of rectilinear figures in numerical cases, generalized to formulae
c) Estimating and measuring areas of regular and irregular figures
d) Area of a square, rectangle, triangle, and parallelogram in numerical cases, generalized to formulae
e) Applications of perimeter and area using measurement and/or calculation
f) Volume of a rectangular solid and capacity of a container developed through activities; estimation and calculation
g) Measurement of the volume of irregular solids using liquid displacement

Algebra (A)

Section 1. Communications
a) Using parentheses
b) Using diagrams to represent mathematical situations
c) Using flow charts in numerical and non-numerical problems

Section 2. Equations
a) Converting word statements to mathematical notation using variables
b) Solving equations in one variable by systematic trial using whole numbers and decimals

Section 3. Relations
a) Representing non-numerical and numerical relationships by arrow diagrams; mappings
b) Representing relations using ordered pairs on a grid

Geometry (G)

Section 1. Basic Notions in Geometry
a) Using geometric terms in correct contexts
b) Intersecting, perpendicular, and parallel lines; properties
c) Symmetries of plane figures including regular polygons; congruence of corresponding parts
d) Classifying angles, triangles, and quadrilaterals

Section 2. Constructions
a) Constructing congruent segments, congruent angles, perpendicular lines, perpendicular bisector, angle bisector, and parallel lines
b) Constructing rectilinear figures
c) Measuring and constructing angles with a protractor; angle properties of various figures

Section 3. Patterns With Tiles
a) Fitting two or more congruent tiles; properties of the figures formed
b) Relating slides, flips, and turns to the figures in (a)
c) Tangram and polyomino puzzles; the symmetries of the tiles and of the figures
d) Tiling the plane
e) Identifying congruent figures, similar figures, line and angle properties in tiling patterns
f) Identifying slides, flips, turns, and symmetries in tiling patterns
g) Area concepts in tiling patterns

Section 4. Transformations
a) Image of a figure under a slide, flip, or turn
b) Translation, reflection, and rotation related to the motions in (a); the fundamental property of each
c) Constructing translation, reflection, and rotation images; congruence of corresponding parts
d) Line symmetry and rotational symmetry as mappings; congruence of corresponding parts
e) Given a figure on a squared grid, drawing its image on a distorted grid

Section 5. Enlargements and Reductions
a) Enlarging and reducing plane figures using geoboards, dot paper, tiles, or grids; properties of shape and size, other properties; similar figures
b) Scale drawings representing real-life situations; scale ratio; converting scale measurements to actual measurements; maps
c) Similar solids

Section 6. Three-dimensional Geometry
a) Identifying real-world objects as solids, shells, or skeletons
b) Polyhedra in the real world; constructing regular polyhedra from their nets
c) Constructing skeletons of pyramids and prisms
d) Sketching 3-D objects
e) Identifying planes of symmetry of 3-D objects

These topics are optional.

Grade 8 Mathematics

8

Numerical Methods (N)

Section 1. Number Applications
a) Operations with whole numbers and decimals; applications
b) Evaluating powers with whole number exponents; applications
c) Numerical development of the rules for multiplication and division with powers
d) Geometric significance of square and cube
e) Roots of perfect squares
f) Using calculators in the above topics; decimal factors

Section 2. Fractions, Ratio, Rate, Percent
a) Computation with fractions; applications
b) Converting fractions to decimals
c) Interpreting scale diagrams by measurement and/or calculation
d) Applying ratio and rate in real-life situations
e) Applying percent in problems related to sales tax and simple interest, and to other topics such as discount, gain or loss, commission, inflation
f) Extended problems involving percent

Section 3. Rational Numbers in Decimal Form
a) Demonstrating that the decimal form of rational numbers either terminates or repeats; converting terminating decimals to fractions
b) Investigating repeating number patterns with a calculator
c) Rounding errors resulting from successive calculations, in particular with calculators
d) Representing large and small numbers in scientific notation

Section 4. Data Graphs †
a) Constructing and interpreting bar and line graphs
b) Population samples with simple attributes expressed in ratio form
c) Extending the tally chart to include percent distributions
d) Representing percent distributions by circle graphs
e) Data distribution expressed as fractions with a sum of 1
f) The mode of the population; modal frequency represented as a percent of the population

Section 5. Integers
a) Establishing the need for integers from practical examples
b) Locating integers on the number line; order
c) Adding and subtracting with integers
d) Multiplying and dividing with integers

Section 6. Measurement of Length, Area, Volume
a) Calculating the perimeter and area of rectilinear figures; applications
b) Investigating the surface area of rectangular solids and other prisms
c) Volume of a rectangular solid and of other prisms, and capacity of a container developed through activities; estimation and calculation

Section 7. The Circle and Related Measures
a) Investigating circular objects to find circumference/diameter, and area/(radius)^2 as an approximation of π; formulae for circumference, area; calculations
b) The surface area of a cylinder related to its net
c) The volume of a cylinder determined by the area of the base and the height

Algebra (A)

Section 1. Formulae and Equations
a) Converting word statements to mathematical notation; concept of variable; substituting in algebraic expressions
b) Developing formulae by generalizing from numerical cases
c) Substituting in formulae and solving resulting equations; applications
d) Solving linear equations in one variable by methods such as inspection, systematic trial, and inverse flow charts
e) Solving simple inequations in one variable by inspection
f) Business formulae related to situations such as interest and commission

Section 2. Relations
a) Non-numerical and numerical relations; arrow diagrams; mappings
b) Representing relations using ordered pairs on a grid
c) Linear relations

Section 3. Probability
a) Listing the possible outcomes of simple experiments
b) Distinguishing the favourable outcomes from the possible outcomes; determining the associated probability
c) Comparison of predicted and experimental results
d) Using sets to represent different attributes within the population
e) Using Venn diagrams to represent and analyse the situations in (d)

Section 4. Flow Charts
a) Representing sequential activities by flow charts
b) Arranging disordered events into logical order
c) Using a flow chart to form an equation such as $3x - 2 = 11$; solution by the inverse flow chart
d) Flow charts of algorithms for calculators

Geometry (G)

Section 1. Constructions
a) Constructing congruent segments and angles, perpendicular lines, parallel lines, perpendicular bisector, angle bisector; tests for parallel and perpendicular lines
b) Constructing a triangle congruent to a given triangle; sufficiency conditions; constructing special cases of triangles and quadrilaterals
c) Locating the centre of a circle and of a circular arc; concentric circles

Section 2. Transformations †
a) Constructing translation, reflection, and rotation images from the fundamental properties
b) Other properties of translation, reflection, and rotation
c) Translation, reflection, and rotation images in the real world
d) Tests to show whether two congruent figures are related by translation, reflection, or rotation; constructing the translation arrow, reflection line, or rotation centre and angle
e) Visual reflection in curved mirrors and flat mirrors; "reflection in a line" as a model of visual reflection in a flat mirror
f) Enlarging and reducing geometric figures using geoboards, dot paper, and/or grids
g) Drawing dilatation images; scale factor; properties
h) Applications of enlargements and reductions in the real world; scale drawings; maps
i) Tiling patterns to illustrate properties of dilatation

Section 3. Properties of Plane Figures
a) Properties of triangles, parallel lines, and quadrilaterals; the Pythagorean Theorem
b) Applying the properties of (a) in numerical exercises
c) Rigidity and non-rigidity of polygonal figures; real-life applications
d) Polygons; properties of regular polygons; symmetry

Section 4. Three-dimensional Geometry
a) Constructing skeletons of pyramids and prisms
b) Sketching 3-D objects
c) Making polyhedra from their nets; stacking solids in space
d) Symmetry of 3-D objects; relationship of planes, axes, and point of symmetry; real-world applications
e) Dilatation images of 3-D objects
f) Reflections in parallel mirrors, in intersecting mirrors; the image in the right-angle corner of two intersecting mirrors

† Full implementation of the section is to begin no later than 1982-83.
These topics are optional.

The Advanced Program for Grades 9 and 10

The advanced program for Grades 9 and 10 is designed for students who:

- have successfully completed Grade 8 Mathematics and acquired a good understanding of the concepts and skills taught in the K-8 program and who can apply this knowledge in a variety of practical and abstract situations;
- plan to study mathematics at the advanced level at least to the end of Grade 12, and possibly to Grade 13 or beyond;
- express interest in university studies or other post-secondary education in which mathematics will be needed at least in an applied role.

Locally developed courses

The advanced program is to consist of courses that are developed locally from the outlines of topics in Charts 4 and 5 (see pages 22 and 23) and that are consistent with the policy and directions set out in this guideline. These courses are to be extensions of the K-8 courses provided in the feeder elementary schools, and should provide a firm basis for those students who wish to study mathematics in Grade 13 or beyond. As with all courses in the Intermediate Division, it is intended that they evolve intuitively from practical experiences and activities with concrete and semi-concrete materials to generalizations and abstractions that are appropriate for the students' mathematical maturity.

Implementation

As discussed in the notes on the program for Grades 7 and 8, the complete K-8 program will not be fully established in the elementary schools until 1983 at the earliest. This in turn will affect the nature of the program that is possible in Grades 9 and 10 during the first few years. The development of the core topics will likely require more instructional time during this period than will be the case once the full program has been implemented.

For this reason, the core topics are being introduced in two stages. Topics that have traditionally been in the program for Grades 9 and 10 are to be part of the new program beginning in 1981-82. The remaining core topics are new to the program and develop sequentially from Grades 7 to 10. They have been coded for delayed implementation and may be cosidered optional in the first few years. During these years, teachers are encouraged to introduce some aspects of these topics by beginning with the necessary background ideas from earlier courses. Full implementation of the new Grade 9 program is to take place no later than 1983-84, and of the Grade 10 program no later than 1984-85.

The Advanced Program

Grade 9 Advanced Mathematics

Section	*Numerical Methods*	*Algebra*	*Geometry*
1	Number Applications 8	Flow Charts	*Constructions, Applications 6
2	*††Statistics 9	*Polynomials 9	*††Coordinate Geometry 4
3	*Powers, Roots 8	Operations With Rational Expressions	*††Congruence Transformations 5
4	*Rational Numbers 7	Equations 13	Dilatation
5	*††Applications of Percent, Ratio, and Measurement 7	*Relations 7	Displacement
6	Lists, Tables	Probability	

Grade 10 Advanced Mathematics

Section	*Numerical Methods*	*Algebra*	*Geometry*
1	Radicals 4	*Equations and Inequations 12	*Coordinate Geometry 10
2	Applied Trigonometry	*Polynomials and Rational Expressions 11	*†††Coordinates and Transformations 6
3	*†††Statistics 10	Relations	*†††Isometries 5
4	Matrices	Probability	*†††Deductive Geometry 15
5		Computers	*Vectors 6
6			*3-D Geometry 4

*Some topics in the section are optional.
All topics in the section are optional.
†† Full implementation of the section is to begin no later than September 1983-84.
††† Full implementation of the section is to begin no later than September 1984-85.

Once students have a full understanding of at least the core topics from all earlier courses, it is expected, under normal circumstances, that 75 percent of the available course time will be required for the development of the core topics. In the above chart, the green numerals beside the sections that include core topics represent a suggested number of hours of instructional time for these core topics.

Using numerical methods
A balance of topics from the three strands is to be developed in Grades 9 and 10. Teachers should consciously apply numerical methods, whenever possible, in the algebra and geometry strands. Flow charts, substitution in algebraic expressions, the solving of equations and formulae by inspection and by systematic trial, relations, probability, geometric properties, computers, coordinate geometry, graphing, dilatation, displacements, coordinates and transformations, mappings, vectors, three-dimensional geometry, matrix representation of transformations, and applications of mathematics all include aspects that can be developed using numerical approaches.

Calculators
Calculators and computers of varying degrees of sophistication are becoming an increasing part of everyone's experience. They are programmed by the human mind, and are essentially appendages of it. They extend our potential to perform calculations involving complicated numerals with ease and to deal with problems and applications otherwise beyond human ability. For these reasons, it is crucial that students learn the capabilities of these devices and the dangers inherent in their misuse. At the same time, the program must recognize that there is still a need for mental and simple written calculations. The human mind is still the world's most valuable "machine" because it is capable of decision-making and logical thought. Thus the school should ensure that every student increases his or her number sense and confidence in working efficiently with numbers to solve problems.

Flexibility in the courses
There will likely be a wide range of abilities and personal needs within each class. Consequently, sufficient flexibility should be built into the courses to meet the needs of gifted students at the one extreme, and of students who are experiencing difficulty in comprehending the material at the other. Although all of these courses will deal with the same core topics, they may differ in depth and breadth of treatment, approach, methodology, activities, applications, selection of options, performance levels expected, and evaluation procedures. It is important that all students be given opportunities to gain a sense of achievement and success from their endeavours.

Continuity in planning
It is important that secondary school teachers be aware of the background of incoming students so that they may assist them with their course selections. Teachers should consult with teachers of feeder schools to ensure that the program flows smoothly from Grade 8 to Grade 9 and evolves from year to year towards the full implementation of all topics by 1983-84. Similarly, Grade 10 teachers should consult with their Grade 9 counterparts in order to establish program continuity.

When a diversity of courses is offered, it is important for the school to keep an accurate record of the nature of the courses and the names of the students taking them, as well as of the performance levels of these students. In this way, unnecessary repetition of topics or gaps in the development of topics can be minimized in successive courses.

Grade 9 Advanced Mathematics

9 Adv

Numerical Methods (N)

Section 1. Number Applications
a) Operations with whole numbers, decimals, percents; adding and subtracting with integers; applications in real-life situations
b) Multiplying and dividing with integers; applications

Section 2. Statistics ††
a) Data-gathering methods such as personal interviews, questionnaires, and measurement activities
b) Random sampling
c) Clustered and stratified sampling
d) Practical experiences in data surveying
e) Organizing data into frequency tables
f) Representing data by graphs and other methods
g) Determining the mode, median, and mean
h) Appropriateness of mode, median, and mean as measures of central tendency
i) Techniques to simplify the calculation of the mean
j) Non-rigorous predictions based upon samples

Section 3. Power, Roots
a) Powers with integral exponents, their evaluation
b) Laws for multiplication and division with powers; numerical and algebraic cases; powers such as $(5^2)^3$
c) Scientific notation; applications in other disciplines
d) Decimal approximations of square roots; applications in real-life situations, including the Pythagorean Theorem
e) Newton's method for calculating square roots
f) Powers with fractional exponents, and decimal exponents

Section 4. Rational Numbers
a) Operations with fractions
b) Reciprocals as multiplicative inverses
c) Converting fractions to decimals; terminating or repeating
d) Converting terminating decimals to fractions
e) Number patterns associated with rational numbers in decimal form; using calculators to form number patterns
f) Errors resulting from the rounding techniques of calculators
g) Irrational numbers as non-terminating, non-repeating decimals; real numbers

Section 5. Applications of Percent, Ratio, Rate, Measurement ††
a) Problems from business and commerce involving percent
b) Reductions and enlargements as percentage changes
c) Measurement and interpretation of scale drawings and models
d) Solving real-world problems using ratio and rate
e) Numerical investigation of compounding situations such as growth, decay, interest
f) Surface area of solids, including pyramids and cones; surface area related to nets
g) Volume of solids, including pyramids and cones; volume relationship between a cone and a cylinder, between a pyramid and a prism

Section 6. Lists, Tables
a) Displaying numerical data by lists and tables; matrices
b) Locating elements by row and column
c) Updating lists and tables

Algebra (A)

Section 1. Flow Charts
a) Flow charting as an organizational procedure
b) Loops for repetitive sequences of operations; applications in solving problems and generating sequences
c) Efficient algorithms for use with calculators

Section 2. Polynomials
a) Substituting in algebraic expressions
b) Adding and subtracting polynomials
c) Multiplying a polynomial by a monomial
d) Common factors
e) Dividing a polynomial by a monomial
f) Multiplying two binomials
g) Polynomials in nested form
h) Solving non-linear equations in one variable by systematic trial, using nested form
i) Factoring trinomials of the form $x^2 + bx + c$, factoring difference of squares

Section 3. Operations With Rational Expressions
a) Reducing rational expressions
b) Multiplying and dividing with rational expressions
c) Adding and subtracting rational expressions with monomial denominators

Section 4. Equations
a) Solving linear equations in one variable by methods such as inspection, systematic trial, and inverse flow charts, and by an algebraic method
b) Linear inequations in one variable; drawing their graphs
c) Solving word problems
d) Developing formulae from diagrams, numerical problems, and measurement activities
e) Substituting in formulae and solving resulting equations; manipulating formulae; applications

Section 5. Relations
a) Representing relations by arrow diagrams, tables, and ordered pairs; mappings
b) Drawing graphs of relations from the real world
c) The linear relation and its graph
d) Solving a system of two linear relations by drawing their graphs, and algebraically

Section 6. Probability
a) Probability related to counting problems in real-life situations
b) Calculating probability using tree diagrams

Geometry (G)

Section 1. Constructions, Applications
a) Basic constructions related to the fundamental property of a circle and to line symmetry; point on a perpendicular bisector; point on an angle bisector
b) Properties related to the circle; applications in numerical cases
c) Concurrent lines of a triangle; circumcircle and incircle; centre of gravity

Section 2. Coordinate Geometry ††
a) Drawing figures on a grid, given the coordinates of their vertices
b) Images of the figures in (a) under translation, reflection, rotation, and dilatation
c) Slope of a segment in numerical cases; applications in the real world
d) Distance between two points using the Pythagorean Theorem for numerical cases; applications
e) Properties of the image of a segment under translation, dilatation, and special cases of reflection and rotation
f) Images of rectilinear figures under vertical and horizontal stretches, and other distortion mappings

Section 3. Congruence Transformations ††
a) Constructing translation, reflection, and rotation images from their fundamental properties; other properties of these transformations, including symmetry
b) Given two congruent figures, testing for the type of transformation; determining the translation arrow, reflection line, or rotation centre and angle
c) Half-turn; properties including point-symmetry; applications
d) Glide-reflection; properties
e) Isometries; meaning of congruence; direct and opposite congruence; determining the isometry for two congruent figures

Section 4. Dilatation
a) Constructing dilatation images with scale factor k when $k \geq 1$, $0 < k < 1$, and $k < 0$
b) Properties of dilatation; ratios of corresponding sides, ratio of corresponding areas; other properties; similar figures
c) Investigating numerically the slope and length of a segment under a dilatation, using coordinates
d) Illustrating the properties of dilatation using tiling patterns
e) Testing to determine whether two figures are related by a dilatation

Section 5. Displacement
a) Representing a displacement by an arrow, successive displacements, and the resultant displacement by arrows
b) Representing a displacement by an ordered pair, successive displacements, and the resultant displacement by ordered pairs
c) Navigation problems involving distance and bearing
d) Investigating travel routes on a globe; intuitive notions of the need for other geometries for world and space travel

†† Full implementation of the section is to begin no later than 1983-84.
These topics are optional.

Grade 10 Advanced Mathematics

10 Adv

Numerical Methods (N)

Section 1. Radicals
a) Entire and mixed radicals
b) Radical factors
c) Operations with radicals (excluding rationalization of denominators); Pythagorean Theorem

Section 2. Applied Trigonometry
a) Activities that relate the angles of right-angled triangles to the ratios of their sides
b) Sine, cosine, and tangent of an angle
c) Solving problems involving right-angled triangles; angles of elevation and depression

Section 3. Statistics †††
a) Experiments that lead to the concept of data forming a distribution
b) Experiments in which statistical measures of samples are compared to the corresponding measures of the population
c) Simple inferences; predictions based on sample polls; reliability of predictions
d) Histogram; distinguishing a bar graph from a histogram; class boundaries for histograms
e) Cumulative frequency tables and curves
f) Deciles; percentiles

Section 4. Matrices
a) Storing information in matrices; dimensions of a matrix
b) Adding matrices
c) Multiplying matrices
d) The identity matrix for multiplication
e) The inverse of a 2×2 matrix
f) Using the inverse matrix in solving pairs of simultaneous equations
g) Matrices and transformations

Algebra (A)

Section 1. Equations and Inequations
a) Drawing the graph of a linear equation, an inequation in two variables
b) Drawing the graphs of two linear equations, two or more inequations; intersection set
c) Solving a system of two linear equations algebraically
d) Solving systems in (c) using matrices
e) Applying the preceding topics in practical situations
f) Linear programming
g) Drawing the graph of linear equations expressed in parametric form

Section 2. Polynomials and Rational Expressions
a) Multiplying polynomials
b) Factoring trinomials of the form $x^2 + bx + c$, factoring difference of squares
c) Reducing rational expressions
d) Multiplying and dividing with rational expressions
e) Adding and subtracting rational expressions with monomial denominators

Section 3. Relations
a) Identifying graphs that represent functions; vertical-line test
b) Drawing the graphs of circles, ellipses, and parabolas from tables of values
c) Recognizing the above figures in architectural designs, and in other art forms

Section 4. Probability
a) Concept of sample space; tree diagrams
b) Probability of occurrence of an event, non-occurrence; experiments
c) Dependent and independent events

Section 5. Computers
a) History and social implications of computers; possible future developments
b) Decision boxes and branching
c) Iterative techniques
d) Studying a computer language
e) Applying flow charts in computer programming, mentally or diagrammatically
f) Using flow charts in solving problems and generating tables
g) Testing programs by numerical methods

Geometry (G)

Section 1. Coordinate Geometry
a) Slope of a segment; applications
b) Slopes of parallel segments, perpendicular segments
c) Distance between two points in numerical cases; formula; applications
d) Midpoint of a segment in numerical cases
e) Illustrating properties of triangles and quadrilaterals using numerical coordinates
f) Investigating real-world situations related to direct variation and partial variation; representing the data graphically
g) Graphing equations of the form $y = x$, $y = mx$, $y = x + b$, $y = mx + b$; the roles of m and b; applications
h) Fitting a straight line to the data in (f)
i) Graphing equations of the form $y = \frac{1}{x}$, $y = \frac{k}{x}$; inverse variation; applications

Section 2. Coordinates and Transformations †††
a) Developing mapping rules for the image of a point (x,y) under translation, special cases of reflection and rotation, and dilatation
b) Investigating a segment and its image under the mappings in (a)
c) Determining the equation of the image of a line under the mappings in (a)
d) Using a 2×2 matrix to find images of points under certain mappings

Section 3. Isometries †††
a) Glide-reflection; properties
b) Isometries; invariant properties; meaning of congruence; direct and opposite congruence; determining the isometry for two congruent figures
c) Composite of reflections in two parallel lines as a translation, and in two intersecting lines as a rotation; fundamental properties of each
d) Composite of three reflections related to glide-reflection
e) Investigating other cases of the composition of two or more isometries

Section 4. Deductive Geometry
a) Geometric theorems and deductions related to congruent figures, parallel lines, and angles of triangles and polygons developed by techniques of traditional Euclidean Geometry, or transformations, or other approaches, or some combination of these
b) Establishing the Pythagorean Theorem using any of a variety of methods

Section 5. Vectors
a) Vector quantity; vectors represented by arrows (directed line segments); equivalent arrows for one vector; vectors as ordered pairs
b) Adding vectors using the Triangle Law
c) Multiplying a vector by a scalar
d) Solving problems related to navigation and force using vectors
e) Deriving geometric properties using vectors
f) Representing vectors in three dimensions by arrows, by ordered triples; the Triangle Law using arrows and/or triples; applications

Section 6. Three-dimensional Geometry
a) Constructing cylinders and cones from their nets; determining the surface area and volume of cylinders, cones, and spheres
b) Packaging of solids and liquids
c) Examples of plane symmetry, rotational symmetry, and point symmetry in the real world
d) Mappings from 3-D to 2-D; projections
e) Other topics from Section 7G-6 and 8G-4

††† Full implementation of the section is to begin no later than 1984-85.
These topics are optional.

The General Program for Grades 9 and 10

The general program is designed for students who:

- have completed Grade 8 Mathematics, but have experienced difficulty in consolidating and applying the skills and concepts of the Grade 7 and 8 program;
- are planning to study Applications of Mathematics courses in the Senior Division;
- do not intend to study mathematics in Grade 13, and are not planning to study mathematics or subjects in which mathematics is required at the university level;
- may be planning to take courses requiring mathematics at a college of applied arts and technology or other post-secondary institution other than a university.

Locally developed courses

The general program is to consist of courses that are developed locally from the outlines of topics in Charts 6 and 7 (see pages 28 and 29) and that are consistent with the policy and directions set out in this guideline. These courses are to be extensions of the K-8 courses provided in the feeder elementary schools, and should reflect the special needs of the students described above. They should evolve from practical experiences and activities with concrete and semi-concrete materials, and should stress useful applications. A foundation should be established for courses in the Applications of Mathematics program in the Senior Division and for possible further studies involving mathematics at a college of applied arts and technology or a post-secondary institution other than a university.

Implementation

As discussed in the notes on the program for Grades 7 and 8, the complete K-8 program will not be fully established in the elementary schools until 1983 at the earliest. This in turn will affect the nature of the program that is possible in Grades 9 and 10 during the first few years in which this guideline is in effect. The development of the core topics will likely require more instructional time during this period than will be the case once the full K-10 program has been implemented.

For this reason, the core topics are being introduced in two stages. Topics that have traditionally been in the program for Grades 9 and 10 are to be part of the new program beginning in 1981-82. The remaining core topics are new to the program and develop sequentially from Grades 7 to 10. They have been coded for delayed implementation and may be considered optional in the first few years. In the intervening years, teachers are encouraged to introduce some aspects of these topics by beginning

The General Program

Grade 9 General Mathematics

Section	*Numerical Methods*	*Algebra*	*Geometry*
1	Number Applications 12	*Problem Solving and Algebraic Strategies 18	*The Circle 5
2	*††Statistics 10	Polynomials	*Coordinate Geometry 3
3	*Powers, Roots 5	Relations	Congruence Transformations
4	*Rational Numbers 7	Probability	*††Dilatation 6
5	Percent 7		Displacement
6	*Applications of Ratio and Measurement 9		

Grade 10 General Mathematics

Section	*Numerical Methods*	*Algebra*	*Geometry*
1	Number Applications 10	*Relations 4	Geometric Figures and Properties 10
2	Ratio and Rate 10	*Linear Equations and Inequations 11	Coordinate Geometry 8
3	†††Statistics 9	*Polynomials 3	Vectors
4	*Square Roots 6		
5	Personal Finance 9		
6	Payrolls and Flow Charts		
7	Applied Trigonometry		
8	Matrices		

* Some topics in the section are optional.
All topics in the section are optional.
†† Full implementation of the section is to begin no later than September 1983-84.
††† Full implementation of the section is to begin no later than September 1984-85.

with the necessary background ideas from earlier courses. Full implementation of the Grade 9 program is to take place no later than 1983-84 and of the Grade 10 program no later than 1984-85.

Timing
Once students have a full understanding of at least the core topics from all earlier courses, it is expected, under normal circumstances, that 75 percent of the available course time will be required for the development of the core topics. In the accompanying chart, the green numerals beside the sections that contain core topics represent a suggested number of hours of instructional time for these core topics.

Comparison of the Grade 9 general and advanced courses
While the section titles in Grade 9 General Mathematics and Grade 9 Advanced Mathematics are almost the same, a study of the outlines of topics for these two courses (see Charts 4 and 6) indicates that students in the advanced-level course are expected to study a more comprehensive core program. In addition, the scope of the classroom development of the advanced course is intended to be more extensive; specifically, it is expected that topics will be treated in greater breadth and depth; that the treatment will include generalizations and some abstractions; that a greater variety of approaches and methodologies will be used; that more sophisticated examples and applications will be chosen; and that expectations for student performance will be of a higher order. Both the general and advanced courses feature concrete activities and an emphasis on relevant real-life applications.

In the general course, students should spend more time at the experiential level and more time assimilating concepts and practising skills. It is unlikely that they will be ready to extend their experiences to the abstraction stage. For example, in developing the concept of "interest", initial experiences may deal with bank accounts, Christmas clubs, lay-away plans, savings bonds, and so on. While students may find it useful to check the interest payments involved in their own personal financial transactions, contrived situations using realistic materials such as bank deposit forms are also effective. Following such activities, consideration could be given to a study of deposits, withdrawals, changes in interest periods, and interest rates. At this stage, the topic of compound interest might be introduced as an example of repeated multiplication.

While students in both the general and advanced courses could consider some of the above numerical applications, students in the advanced course could be expected to generalize these ideas into the formula $A = P(1 + i)^n$, and then to look at other situations of a similar mathematical nature, such as the study of population growth and carbon-dating.

Transfer of students from one level to the other
The Grade 9 general and advanced courses are designed to accommodate the transfer of a student from one level to the other with a minimum of disruption in the student's program. The parallel design of these two courses also makes it possible to modify the program of individual students during the year, when a mid-year transfer is not advisable or feasible. The above plan applies only to Grade 9 courses; it is assumed that by Grade 10 most students will have settled on the level of study that is appropriate for their personal needs. The Grade 10 general course has a stronger applications orientation than has the Grade 10 advanced course.

Adapting the Grade 10 general course for business and technical programs
Through the selection of options, it is possible to adapt the Grade 10 General Mathematics course for students who have an interest in business subjects or those who have an interest in technical subjects. If conditions within the school warrant it, a Grade 10 course entitled "Grade 10 General Mathematics, With Business Applications" may be established. This course should be based on the outline of topics for Grade 10 General Mathematics. The core topics should be developed within the context of business applications, whenever possible, and the optional topics selected should have a business orientation. Optional topics other than those listed in this guideline may be used at the discretion of the teacher, but these should not significantly overlap business topics to be developed in Grades 11 and 12. A course entitled "Grade 10 General Mathematics, With Technical Applications" may also be developed, subject to similar conditions.

Using numerical methods
A balance of topics from the three strands is to be developed in Grades 9 and 10. Since many of the students will proceed to studies at colleges of applied arts and technology, it is important that essential concepts and skills of arithmetic, algebra, and geometry be understood and applied. In particular, emphasis should be placed on the application of numerical methods in solving algebraic and geometric problems. Flow charts, substitution in algebraic expressions and formulae, the solving of equations by inspection and by systematic trial, relations, probability, geometric properties, coordinate geometry, graphing, dilatation, displacement, and vectors all have aspects that can be investigated using numerical methods. The comments made concerning the use of calculators in this guideline (see p. 11) are especially applicable to numerical methods in this program.

Flexibility in the courses

Students in the general program will exhibit a variety of strengths and weaknesses. Thus it is important that courses be sufficiently flexible to accommodate these varying needs. Whenever possible, "hands-on" activities and practical applications relevant to the students' personal experiences should be used. Ample time should be allowed for establishing essential concepts and skills. Extensions of these, in terms of breadth, depth, variety of approaches, and the sophistication of the applications and abstractions, should be treated cautiously at this stage, and should be attempted only with those students who appear ready for them. Above all, it is important that the program be interesting for the students and that it provide them with opportunities to complete the work successfully. In this way, their self-confidence will be enhanced, and they will approach mathematics with greater enthusiasm.

Continuity in the program

It is important that secondary school teachers be aware of the background of incoming students so that they may assist them with their course selections and adapt courses to their specific needs. Teachers should consult with teachers of feeder schools to ensure that the program flows smoothly from Grade 8 to Grade 9 and evolves from year to year towards the full implementation of all topics by 1983-84. Similarly, Grade 10 teachers should consult with their Grade 9 counterparts in order to establish program continuity.

As in the advanced program, where a diversity of courses is offered, it is important for the school to keep an accurate record of the nature of the courses offered and the names of the students taking them, as well as of the performance levels of these students. In this way, unnecessary repetition of topics or gaps in the development of topics can be avoided in successive courses.

Grade 9 General Mathematics

9 Gen

Numerical methods (N)

Section 1. Number Applications
a) Operations with whole numbers, decimals, and percents; adding and subtracting with integers; applications in real-life situations
b) Multiplying and dividing with integers; applications

Section 2. Statistics ††
a) Appropriateness of data; data from primary and secondary sources; data-gathering methods; quantitative and qualitative data
b) Constructing, reading, and interpreting tally charts and frequency tables
c) Displaying data using pictographs, bar graphs, circle graphs, and line graphs; reading and interpreting graphs; choosing the appropriate type of graph
d) Determining the mode, median, and mean
e) Appropriateness of the mode, median, and mean as measures of central tendency

Section 3. Powers, Roots
a) Evaluating powers with whole number exponents; applications
b) Decimal approximations of square roots; applications in real-life situations
c) Newton's method for calculating square roots
d) Using the Pythagorean Theorem in practical situations
e) Powers with integral exponents, evaluating them
f) Laws for multiplying and dividing with powers developed from numerical cases; powers such as $(5^2)^3$
g) Scientific notation; applications in other disciplines

Section 4. Rational Numbers
a) Operations with fractions; reciprocals
b) Rational numbers in decimal form
c) Number patterns associated with rational numbers in decimal form; using calculators to form number patterns

Section 5. Percent
a) Consolidating and extending the concept of percent; fraction, decimal, percent conversions
b) Fractional percents; percents greater than 100%
c) Applications in the real world

Section 6. Applications of Ratio, Rate, Measurement
a) Using ratio and rate concepts in real-world situations
b) Measuring and interpreting scale drawings, maps, and physical models
c) Numerical investigation of situations involving growth and decay
d) Surface area of solids, including pyramids and cones; surface area related to nets
e) Volumes of solids, including pyramids and cones; volume relationship between a cone and a cylinder; between a pyramid and a prism

Algebra (A)

Section 1. Problem Solving and Algebraic Strategies
a) Using diagrams and/or flow charts to organize information in solving problems
b) Forming linear equations in one variable; solving the equations by methods such as inspection, systematic trial, and inverse flow charts, and by an algebraic method
c) Linear inequations in one variable; drawing their graphs
d) Repetitive sequences of operations; applications to solving problems
e) Algorithms for use with calculators
f) Developing formulae from diagrams, numerical problems, and measurement activities
g) Substituting in formulae and solving resulting equations; applications
h) Solving word problems

Section 2. Polynomials
a) Substituting in algebraic expressions
b) Adding and subtracting polynomials
c) Multiplying a polynomial by a monomial
d) Common factors

Section 3. Relations
a) Representing relations by arrow diagrams, tables, and ordered pairs
b) Graphing relations from the real world
c) The linear relation and its graph

Section 4. Probability
a) Probability related to counting problems in real-life situations
b) Calculating probability using tree diagrams

Geometry (G)

Section 1. The Circle
a) Basic constructions and their applications
b) Concurrent lines of a triangle; circumcircle and incircle; centre of gravity
c) Investigating properties related to a circle
d) Solving numerical problems using the properties of (b) and (c)

Section 2. Coordinate Geometry
a) Drawing figures on a grid, given the coordinates of their vertices
b) Images of the figures in (a) under translation, reflection, rotation, and dilatation
c) Given a figure on a squared grid, drawing its image on a distorted grid
d) Experimenting with inclined planes; the notion of slope; applications in the real world
e) Distance between two points using the Pythagorean Theorem in numerical cases: applications

Section 3. Congruence Transformations
a) Constructing translation, reflection, and rotation images from their fundamental properties; other properties of these transformations, including symmetry
b) Half-turn; properties, including point symmetry; applications

Section 4. Dilatation ††
a) Constructing dilatation images on grids with scale factor k when $k \geq 1$, $0 < k < 1$
b) Ratios of corresponding sides; ratio of corresponding areas; other properties; similar figures
c) Applications of enlargements and reductions in the real world; scale drawings
d) Recognizing similar figures and their properties in tiling patterns

Section 5. Displacement
a) Representing a displacement by an arrow; successive displacements and the resultant displacement by arrows
b) Representing a displacement by an ordered pair, successive displacements, and the resultant displacement by ordered pairs
c) Navigation problems using distance and bearing

†† Full implementation of the section is to begin no later than 1983-84.
These topics are optional.

Grade 10 General Mathematics

10 Gen

Numerical Methods (N)

Section 1. Number Applications
a) Consolidating number skills using games, puzzles, and activities; applications in real-life situations
b) Rounding of whole numbers and decimals; applications in real-life situations
c) Approximating and estimating
d) Interpolating and extrapolating from tabulated data
e) Powers with integral exponents, evaluating them
f) Scientific notation; applications in other disciplines

Section 2. Applications of Ratio, Rate
a) Practical examples of ratios with two or more terms
b) Proportions with two-term ratios and three-term ratios; practical examples
c) Practical examples of rate
d) Direct variation; graphs and practical examples
e) Partial variation; graphs and practical examples
f) Inverse variation; graphs and practical examples

Section 3. Statistics †††
a) Critical analysis of statistical statements, tables, and graphs; effect of changing the scale of a graph
b) Reasons for sampling; importance of a good sample
c) Using samples to estimate the composition of a population and its characteristics
d) Estimating the population by using samples of different sizes, by combining the data from many samples
e) Identifying the possible outcomes in experiments; using tallies to record outcomes; drawing the graph of the frequencies of the outcomes
f) Using relative frequencies to predict the outcomes of an experiment

Section 4. Square Roots
a) Decimal approximations of square roots
b) Newton's method for calculating square roots
c) Applications of the Pythagorean Theorem
d) Substituting in formulae involving square roots, simplifying the result; practical applications

Section 5. Personal Finance
a) Checking personal income records
b) Cost of credit
c) Instalment purchases; leases; deferred payment schemes
d) Savings and lending institutions
e) Automobile ownership

Section 6. Payrolls and Flow Charts
a) Organizing calculations using flow charts
b) Generating payroll summaries using flow charts

Section 7. Applied Trigonometry
a) Activities that relate the angles of right-angled triangles to the ratios of their sides
b) Sine, cosine, and tangent of an angle
c) Solving problems involving right-angled triangles; angles of elevation and depression

Section 8. Matrices
a) Storing numerical data in matrix form
b) Adding matrices representing real-world situations
c) Multiplying matrices representing real-world situations

Algebra (A)

Section 1. Relations
a) Numerical relations as sets of ordered pairs
b) Drawing graphs of numerical relations derived from experiments and realistic situations; interpolating and extrapolating from the graphs
c) Drawing graphs of non-linear relations
d) Identifying graphs that represent functions; vertical-line test

Section 2. Linear Equations and Inequations
a) Investigating the roles of m and b in equations of the form $y = mx + b$
b) Drawing the graphs of pairs of equations of the form $y = mx + b$ to determine the intersection; the intersection as the solution
c) Solving real-world problems that can be represented by linear equations
d) Solving a system of two linear equations algebraically
e) Drawing the graphs of linear inequations; boundaries and region of solution

Section 3. Polynomials
a) Substituting in algebraic expressions
b) Adding and subtracting polynomials
c) Multiplying a polynomial by a monomial
d) Common factors
(e) Squaring a binomial

Geometry (G)

Section 1. Geometric Figures and Properties
a) Circle constructions, patterns and properties; applications
b) Properties related to parallel lines, perpendicular lines, triangles and quadrilaterals, and the Pythagorean Theorem; numerical exercises and applications

Section 2. Coordinate Geometry
a) Experimenting with inclined planes; the notion of slope; applications in the real world
b) Slopes of parallel segments, perpendicular segments
c) Distance between two points using the Pythagorean Theorem in numerical cases; applications

Section 3. Vectors
a) Vectors represented by arrows (directed line segments); equivalent arrows for one vector; vectors as ordered pairs
b) Adding vectors using the Triangle Law
c) Multiplying a vector by a scalar
d) Solving problems related to navigation and force using vectors

††† Full implementation of the section is to begin no later than 1984-85.
These topics are optional.

The Basic Program for Grades 9 and 10

The basic program for Grades 9 and 10 is designed for students who:

- have been promoted to Grade 9 on the basis of their overall achievement in Grade 8, but who have not completed the regular mathematics program for Grades 7 and 8;
- require additional experiences with the concepts, skills, and applications of Grade 7 and 8 Mathematics before taking other mathematics courses in the basic program or in a program at a higher level of difficulty;
- may not be planning to study mathematics beyond the two credits in mathematics required in the first two years of secondary school;
- may be planning to enter the work world directly after leaving secondary school;
- may be planning to continue their education after secondary school in fields that do not require further courses in mathematics.

Most students who will take Grade 9 Basic Mathematics will have experienced frustration in their attempts to learn mathematics in the past to the point where many of them may feel defeated even before beginning the course. Although they represent a wide spectrum of abilities, they share a lack of motivation to learn mathematics, and many of them experience anxiety at the prospect of another course in mathematics – this time in the strange, new environment of secondary school. The approach used should stress attention to individual differences, provide opportunities for success, and alleviate student anxiety.

Locally developed courses

The basic program is to consist of courses that are developed locally from the outlines of topics in Charts 8 and 9 (see pages 32 and 33) and that are consistent with the policy and directions set out in this guideline. Even though these courses are based on topics found in Grade 7 Mathematics and Grade 8 Mathematics, their development should not parallel that of the earlier courses. For most students, such an approach would only confirm their lack of success with these ideas in the past. Rather, the concepts and skills should be developed and applied in new contexts.

Students' awareness of the need for mathematical knowledge and skills should be developed through discussions of real-world situations that are of interest to them. These situations will serve a double purpose: they will motivate students by providing them with examples of where and how mathematics is used, and they will help determine the remediation needed by indicating the strengths and weaknesses of individual students as they work with the associated mathematics. The program must provide students with opportunities to experience success if their interest and enthusiasm are to be maintained. Evaluation will need to be frequent, varied, objective *and* subjective, and, above all, positive and encouraging for the students.

Implementation

Implementation of the Basic Mathematics program may begin in 1980, since the core topics in it have been a part of the Grade 7 and 8 courses for many years.

The major mathematical ideas for these courses are shown in the following chart.

The Basic Program

Grade 9 Basic Mathematics

Section	*Numerical Methods*	*Algebra*	*Geometry*
1	*The Counting Numbers	*Problem Solving	Basic Notions of Geometry
2	Computation With Whole Numbers and Decimals	Introduction to Computing	Constructions
3	Fractions	The Mathematics of Chance	Transformations
4	Ratio and Rate		3-D Geometry
5	Percent		
6	Data Presentation		
7	Powers and Roots		
8	Integers		
9	Measurement		

Grade 10 Basic Mathematics

This course is based on the topics of Grade 9 Basic Mathematics, developed in the context of relevant applications.

It is suggested that these applications be developed from themes such as those outlined in the following resources:

a) *Grade 10 Basic Mathematics, Resource Module 4: Themes and References for Their Development*
b) *Grade 9 Basic Mathematics, Resource Module 2: Themes*
c) *Grade 9 Basic Mathematics, Resource Module 3: Mathematics and the Car*

Other themes may be developed locally.

The themes and/or subthemes used in Grade 10 Basic Mathematics should not duplicate those used in Grade 9 Basic Mathematics.

* Some topics in the section are optional.
All topics in the section are optional.

The courses in this program are meant to satisfy the basic mathematical needs of individual students. For this reason, they should be flexible. Since the same skills and concepts will surface in a variety of situations throughout the year, students will have many occasions to recall, practise, consolidate, and apply the same fundamental ideas in entirely different contexts. Thus the extent to which each topic should be studied at a given time can best be determined in the classroom, and no time suggestions are given for individual sections.

The core of Grade 9 Basic Mathematics includes the mathematical topics that students will require for further studies in Grade 9 General Mathematics or in Grade 10 Basic Mathematics. When planning a Grade 9 Basic Mathematics course, teachers should identify where and how these topics are related to the practical situations that are to be investigated, and then establish strategies for developing the core concepts and skills within the context of these situations. However, because of their highly individual needs, all students need not study the topics in the same depth or breadth. The course should be adapted to respond to the individual needs of each student. For this reason, in this course the core topics listed are to be taken as suggestions only.

The concepts and skills needed for the activities in Grade 10 Basic Mathematics are the same as those that are listed for the Grade 9 course. Here also, these core listings should be viewed as suggestions only. Remediation and/or consolidation of these concepts and skills should reflect the individual needs that are identified as students come to grips with the situations being investigated. In this way, students will strengthen their understanding of basic mathematical ideas and their ability to apply them.

The Grade 10 course emphasizes the investigation of situations that most people will confront in their lifetime and focuses on where and how mathematics is used in dealing with these situations. It is designed to prepare students for making personal decisions in practical matters such as budgeting, credit buying, home ownership, insurance, and travel, and to give them a better appreciation of the use of mathematics in recreational areas such as sports and music. Alternately, the practical situations that are investigated may involve applications of mathematics that are related to trade, technical, or business subjects.

The mathematics in Grade 9 and 10 Basic Mathematics courses should be presented in simple form, at a relaxed pace, and in a mode befitting the social maturity of the students. The essential skills and concepts of the program should be based on the students' personal experiences, should make use of materials to which they can personally relate, and should be tied to applications that are real to them now or that they can accept as useful for the future. In this way, the development of mathematical ideas can be intrinsically interesting for the students and may be used to increase their motivation to study the subject.

Resource modules

The Notes for Teachers for Grade 9 Basic Mathematics (Resource Module 1) are intended to help teachers design courses that meet the above criteria. They suggest an expanded development for each topic, and list related topics, concepts, and skills. The other resource modules will help teachers design courses with thematic approaches. Resource Module 2 indicates fifteen themes, and many subthemes within them, through which the concepts and skills of the course can be identified and developed. Resource Module 3 provides a sample development of a subtheme, ready for classroom use. During the classroom development of the themes, a number of applications of mathematics should emerge. When necessary, mini-lessons on specific concepts and skills should be provided for students who need help in these areas. Review materials and practice exercises should be developed and used to consolidate fundamental ideas.

The Notes for Teachers for Grade 10 Basic Mathematics (Resource Module 4) provide nine suggested themes and subthemes. These represent situations that the students have experienced or will experience in adult life. They involve applications of the mathematical skills commonly required in the work world, in the market place, and in the home. Resource Module 5 provides a list of materials that are available from financial institutions in Ontario. These will be useful in developing Theme 3 of the Notes for Teachers for this course.

Continuity in the program

It is important that teachers of these courses keep an accurate record of the themes and practical experiences that are investigated, as well as of the strengths and weaknesses of each student, on a year-by-year basis. In this way, unnecessary repetition of themes and applications can be avoided in successive courses. Whenever possible, Grade 9 teachers should discuss the background of incoming students with teachers of the feeder elementary schools. In order to maintain continuity in the program, Grade 10 teachers should consult with their counterparts in Grade 9.

Grade 9 Basic Mathematics

9 Basic

Numerical Methods (N)

Section 1. The Counting Numbers
a) Counting; numerals and place value
b) Reading numerals, rewriting numerals in words
c) Special numerical terms; applications
d) The concepts of zero and one
e) Ordering and sequencing
f) The notion of a code; coding and decoding

Section 2. Computation With Whole Numbers and Decimals
a) Rounding
b) Adding, subtracting, multiplying, and dividing with whole numbers
c) Operations with decimals
d) Calculations related to money

Section 3. Fractions
a) The concept of fraction
b) Equivalent fractions
c) Reducing fractions
d) Converting a fraction to a decimal
e) Multiplying and dividing with fractions
f) Common denominator; adding and subtracting with fractions

Section 4. Ratio, Rate
a) The concept of ratio; equivalent ratios
b) The concept of rate

Section 5. Percent
a) The concept of percent
b) Converting percents to decimals, decimals to percents
c) Computation with percents

Section 6. Data Presentation
a) Reading tables
b) Reading pictographs
c) Understanding bar, line, and circle graphs
d) Collecting data; descriptive data; numerical data; organizing data
e) Drawing pictographs, bar graphs, line graphs, circle graphs

Section 7. Powers and Roots
a) The meaning of power notation
b) Square root as the inverse of square

Section 8. Integers
a) The concept of integer
b) The order of integers; the integer line
c) Operations with integers

Section 9. Measurement
a) Using appropriate units
b) Estimating and measuring lengths of line segments and curves, using the units mm, cm, m, km
c) Estimating and measuring areas of regular and irregular figures, using the units cm^2, m^2, km^2, ha; calculating
d) Volumes of rectangular solids, using the units cm^3, m^3; calculating volumes
e) Estimating and measuring capacities of containers, using the units mL, L
f) Estimating and measuring mass, using the units g, kg, t

Algebra (A)

Section 1. Problem Solving
a) Analysing and solving problems
b) Aids to problem solving; understanding mathematical words, diagrams, charts and tables, formulae, flow charts, calculators
c) Concept of variable, forming equations, solving equations

Section 2. Introduction to Computing
a) Uses of a computer
b) Working with a computer
c) Binary system
d) Uses of flow charting

Section 3. The Mathematics of Chance
a) Probability in simple two-outcome experiments
b) Probability in experiments with many possible outcomes
c) Sample space by listing
d) Sample space by a tree diagram
e) Size of sample space by the Fundamental Counting Principle
f) Sampling for composition
g) Probability based on experiments
h) Odds

Geometry (G)

Section 1. Basic Notions of Geometry
a) The language of geometry
b) Intersecting, perpendicular, and parallel lines
c) Symmetry
d) Properties of plane figures

Section 2. Constructions
a) Using a protractor
b) Accurate constructions using a variety of instruments

Section 3. Transformations
a) Slides, turns, and flips
b) Enlargements and reductions
c) Distortions

Section 4. Three-dimensional Geometry
a) Solids, shells, and skeletons
b) Polyhedra
c) Pyramids and prisms
d) Sketching 3-D objects

These topics are optional.

Grade 10 Basic Mathematics

Section 1. Outline of Topics

This course is intended to further reinforce and develop the topics of Grade 9 Basic Mathematics. However, the core-option designation in the Grade 9 course does not apply here; the selection of topics will depend on local conditions and on the specific needs of the students. Further, the scope of topics may be extended beyond that of the Grade 9 course, if this is considered desirable.

The course should stress applications of mathematics that are relevant for the students now and will be useful to them in the future. To this end, it is recommended that the program be developed from themes that involve applications of skills and concepts introduced in earlier courses.

Section 2. Outline of Themes

The following themes* are suggested as a basis for developing this course. This list may be extended by adding themes from the list for Grade 9 Basic Mathematics or other locally developed themes.

Theme 1 – Math at Work and at Home
Theme 2 – Your Earnings and Your Budget
Theme 3 – You and Financial Services
Theme 4 – You and Taxes
Theme 5 – You and Travelling — Public and Private
Theme 6 – You and Insurance
Theme 7 – Mathematics in Sports
Theme 8 – Setting Up a Place of One's Own
Theme 9 – You and the Music World

*See the resource modules for Grade 10 Basic Mathematics and Grade 9 Basic Mathematics.

The Modified Program for Grades 9 and 10

The Modified Program for Grades 9 and 10 contains courses at two levels of difficulty. The level 1 program is designed for students who have an understanding of the mathematics in the Primary or early Junior Division program. The level 2 program is designed for students who have an understanding of the mathematics in the Junior Division program.

Locally developed courses

The program for Grades 9 and 10 is illustrated below.

	Level 1	Level 2
Grade 9	Course A	Course B
Grade 10	Course B	Course C

The major mathematical ideas for these courses are summarized in the chart below. A more detailed description is given in the outlines of topics in Charts 10, 11, and 12 (see pages 36-38). The courses are to be consistent with the policy and directions set out in this guideline.

Students in this program generally have difficulty in learning and maintaining the basic mathematical concepts and skills needed for other subjects and for everyday living. To help them consolidate these fundamental ideas, courses A, B, and C all involve essentially the same mathematical topics. However, the contexts in which these topics are identified and applied and the breadth and depth of their development should be different in each course. In this way, students will study new themes and new applications each year and, in this sense, will be exploring new and interesting practical ideas. At the same time, they will be recalling and using the same basic mathematical ideas. When necessary, these ideas should be reviewed with the students who need this help, and the requisite skills practised. The classroom environment should not be threatening to the students; realistic expectations should be set for them so that they will be encouraged by the likelihood of success.

Implementation

Although the core topics listed for these courses should be viewed as suggestions only, they should all be considered during the planning stage for each course. At the same time, because of the highly individual needs of the students in this program and the wide variations in their mathematical ability, it is unrealistic to assume that they are all ready for all topics in the core program or that they will necessarily study all of them in any one course.

Locally planned courses should be flexible so that they can be adapted day by day to the identified

The Modified Program

Modified Mathematics – Course A
Level 1, Grade 9

Section	*Numerical Methods*	*Geometry*
1	*Non-computational Use of Numbers	Motion Geometry
2	*Computational Use of Whole Numbers	*Geometric Constructions
3	Computational Use of Decimals	Prisms and Pyramids
4	Percent	
5	Aids to Solving Mathematical Problems	
6	*Fractions	
7	*Data Graphs	
8	*Measurement	
9	Probability	

Modified Mathematics – Course B
Level 1, Grade 10
Level 2, Grade 9

Section	*Numerical Methods*	*Geometry*
1	Non-computational Use of Numbers	*Motion Geometry
2	Computational Use of Whole Numbers	*Geometric Constructions
3	Computational Use of Decimals	Prisms and Pyramids
4	*Percent	
5	Aids to Solving Mathematical Problems	
6	Fractions	
7	*Data Graphs	
8	*Measurement	
9	*Probability	

Modified Mathematics – Course C
Level 1, Grade 11
Level 2, Grade 10

Section	*Numerical Methods*	*Geometry*
1	Non-computational Use of Numbers	*Motion Geometry
2	Computational Use of Whole Numbers	*Geometric Constructions
3	Computational Use of Decimals	Prisms and Pyramids
4	*Percent	
5	Problem Solving	
6	Fractions	
7	*Data Graphs	
8	*Measurement	
9	*Probability	

* Some topics in the section are optional.
All topics in the section are optional.

needs of individuals or groups of students within each class. Thus a timing schedule is not suggested for the core sections.

Course A for Grade 9, Level 1, and Course B for Grade 9, Level 2, become official in 1981-82. Course B for Grade 10, Level 1, and Course C for Grade 10, Level 2, become official in 1982-83.

Relevant applications
The school has a major responsibility for ensuring that students learn to use mathematics in situations that are relevant for them now and in the future. Thus each course should stress applications that the students see as important and useful. This emphasis may require the collection, organization, and development of special materials, as well as the identification and development of contexts that students can readily recognize as practical and relevant. The development of materials can be undertaken by a number of boards working co-operatively, by individual boards, by groups of teachers, or by individual teachers.

The use of themes that are within the students' range of experiences generates interest and provides motivation for the students and shows them how mathematics is used on a day-to-day basis in the real world. Such themes also provide a natural vehicle for involving students in problem-solving techniques, and for identifying those students who need extra help with their basic skills and/or conceptual understandings.

Resource modules
The Notes for Teachers for courses A, B, and C (Modified Mathematics, Resource Modules 1, 2, and 3) suggest some contexts for developing topics. These should be considered as the basis for developing an extended list of contexts. The Notes for Teachers also identify other topics, concepts, and skills that are related to the topics under discussion. These may be used at appropriate points in the development of each topic. They provide opportunities to recall and consolidate past experiences and to introduce new ideas as the course progresses.

Continuity in the program
Comprehensive records of the work attempted by each student, as well as a description of the contexts in which the work was developed, are of value to teachers of subsequent courses. These records will help in the planning of courses that build upon earlier experiences and that enable students to progress according to their individual ability. Teachers of Grade 9 courses should give careful consideration to the elementary school records of their students. Similarly, teachers of Grade 10 courses should consult with their counterparts in Grade 9.

Modified Mathematics — Course A
Level 1, Grade 9

Numerical Methods (N)

Section 1. Non-computational Use of Numbers
a) Counting from 1 to 100 by 1, 2, 5, 10, 20, 25
b) Writing numbers from 1 to 100 using numerals, words
c) Reading numbers to 1 000 000
d) Arranging numbers in order, smallest to largest; largest to smallest
e) Ranking things as first, second, third, etc., smallest to largest, largest to smallest
f) Simple codes; identifying parts of a code; coding and decoding

Section 2. Computational Use of Whole Numbers
a) Place value: units, tens, hundreds, thousands
b) Adding pairs of numbers with up to three digits
c) Subtracting pairs of numbers with up to two digits
d) Multiplying one- and two-digit numbers by one-digit numbers
e) Factoring numbers, e.g., $6 = 2 \times 3$
f) Dividing by 2, 3, 4, 5, 6, and 10
g) Properties of addition and multiplication, developed informally
h) Reasonableness of answers

Section 3. Computational Use of Decimals
a) Place value from hundreds to hundredths
b) Adding and subtracting pairs of decimals of the types in (a)
c) Multiplying and dividing by 10, 100, 1000
d) Reasonableness of answers
e) Approximating

Section 4. Percent
a) Percent as hundredths, e.g., $45\% = \frac{45}{100} = 0.45$
b) Finding a percent of a number; applications in real-life situations

Section 5. Aids to Solving Mathematical Problems
a) Using simple flow charts
b) Mathematics as a language; common symbols in mathematics; simple sentences; simple formulae
c) Using a calculator

Section 6. Fractions*
a) Fraction as part of a whole, as part of a group
b) Adding and subtracting fractions with the same denominators
c) Multiplying a whole number by a fraction, a fraction by a fraction
d) Decimal equivalents of fractions such as $\frac{1}{2}$, $\frac{1}{5}$, $\frac{1}{4}$, and $\frac{1}{10}$

* simple cases only, with denominators of 2, 4, 8, 3, 5, 10

Section 7. Data Graphs
a) Reading, comparing, and interpreting information from charts and graphs
b) Collecting and organizing data
c) Representing data by pictographs and bar graphs

Section 8. Measurement*
a) Informal measurement of length, area, volume, capacity, and mass using non-standard units
b) Measuring length, estimating
c) Measuring area, estimating
d) Measuring volume, estimating
e) Measuring capacity, estimating
f) Measuring mass, estimating
g) Measuring temperature, estimating
h) Selecting the appropriate unit; equivalent units and conversion

* using metric units cm, m, km, cm^2, m^2, ha, cm^3, m^3, mL, L, g, kg, t, °C, kPa, and imperial units until they are no longer in common use

Section 9. Probability
a) Counting the outcomes of experiments
b) Formula for simple probability

Geometry (G)

Section 1. Motion Geometry
a) Drawing a slide image on a grid; identifying slide images on grids and in the real world
b) Drawing a flip image on a grid; identifying flip images on grids and in the real world; symmetry
c) Quarter, half, three-quarter and full turns; drawing a turn image on a grid; identifying turn images on grids and in the real world; symmetry

Section 2. Geometric Constructions
a) Using a protractor to measure and to construct angles such as 90°, 45°, 60°, 30°; estimating the size of angles
b) Constructing circles using a compass; centre, radius, diameter
c) Constructing triangles using a variety of techniques; right-angled, isosceles, equilateral, scalene triangles
d) Using a compass and protractor to construct a square, equilateral triangle, and other regular polygons in a circle; symmetry
e) Tiling patterns; tiling with triangles, quadrilaterals, regular hexagons

Section 3. Prisms and Pyramids
a) Identifying prisms by the base; vertex, edge, face; description of prisms
b) Identifying pyramids by the base; vertex, edge, face; description of pyramids

These topics are optional.

CHART 11

Modified Mathematics — Course B
Level 1, Grade 10; Level 2, Grade 9

B Mod

Numerical Methods (N)

Section 1. Non-computational Use of Numbers
a) Counting from 1 to 1000 by 1, 2, 5, 10, 20, 25
b) Writing numbers from 1 to 1000 using numerals, words
c) Reading numerals
d) Arranging numbers in order; using numbers to sequence things
e) Ranking things as first, second, third, etc.
f) Simple codes; identifying parts of a code; coding and decoding

Section 2. Computational Use of Whole Numbers
a) Place value; units, tens, hundreds, thousands
b) Adding pairs of numbers with up to four digits
c) Subtracting pairs of numbers with up to four digits
d) Multiplying one- to three-digit numbers by one- and two-digit numbers; squaring numbers 1, 2, 3, . . . , 10
e) Factoring one- and two-digit numbers; perfect squares
f) Dividing by one- and two-digit numbers
g) Properties of addition and multiplication, developed informally
h) Reasonableness of answers
i) Approximating
j) Estimating

Section 3. Computational Use of Decimals
a) Place value from thousands to thousandths
b) Adding and subtracting decimals of the type in (a)
c) Multiplying pairs of decimals of the type in (a)
d) Dividing a decimal of the type in (a) by a one-digit number
e) Multiplying and dividing a decimal of the type in (a) by 10, 100, and 1000
f) Reasonableness of answers
g) Approximating
h) Estimating

Section 4. Percent
a) Meaning of percent
b) Finding a percent of a number; applications in real-life situations
c) Finding the percent of a part to the whole; applications in real-life situations

Section 5. Aids to Solving Mathematical Problems
a) Using simple flow charts
b) Mathematics as a language; common symbols in mathematics; simple sentences; simple formulae
c) Using a calculator

Section 6. Fractions*
a) Fraction as part of a whole, as part of a group
b) Equivalent fractions (simple cases only)
c) Adding and subtracting fractions with the same denominators
d) Multiplying a whole number by a fraction, a fraction by a fraction
e) Decimal equivalents of fractions such as $\frac{1}{2}$, $\frac{3}{5}$, $\frac{3}{4}$, and $\frac{8}{10}$

* with denominators of 2, 4, 8, 3, 5, 10

Section 7. Data Graphs
a) Reading, comparing, and interpreting information from charts and graphs
b) Collecting and organizing data from real-life situations, from experiments
c) Representing data by pictographs, bar graphs, and line graphs
d) Graphing on a coordinate grid

Section 8. Measurement*
a) Informal measuring of length, area, volume, capacity, and mass using non-standard units
b) Measuring length, estimating
c) Measuring area, estimating
d) Measuring volume, estimating
e) Measuring capacity, estimating
f) Measuring mass, estimating
g) Measuring temperature, estimating
h) Selecting the appropriate unit; equivalent units and conversion

* using metric units cm, m, km, cm^2, m^2 ha, cm^3, m^3, mL, L, g, kg, t, C, kPa, and imperial units until they are no longer in common use

Section 9. Probability
a) Counting the outcomes of experiments
b) Formula for simple probability
c) Finding the probability using a tree diagram

Geometry (G)

Section 1. Motion Geometry
a) Drawing a slide image on a grid, on plain paper; identifying slide images in diagrams and in the real world
b) Drawing a flip image on a grid, on plain paper; identifying flip images in diagrams and in the real world; symmetry
c) Quarter, half, three-quarter and full turns; drawing a turn image on a grid, on plain paper; identifying turn images in diagrams and in the real world; symmetry
d) Distortions; drawing a distortion image using a distorted grid

Section 2. Geometric Constructions
a) Using a protractor to measure and construct angles from 0° to 90°; estimating the size of angles
b) Constructing circles using a compass; centre, radius, diameter
c) Constructing triangles using a variety of techniques; right-angled, isosceles, equilateral, scalene triangles
d) Constructing squares and rectangles using a variety of techniques
e) Using a compass and protractor to construct a square, equilateral triangle, and other regular polygons in a circle; symmetry
f) Tiling patterns; tiling with triangles, quadrilaterals, regular hexagons; symmetry

Section 3. Prisms and Pyramids
a) Identifying prisms by the base; vertex, edge, face; description of prisms; examples of prisms in the real world
b) Identifying pyramids by the base; vertex, edge, face; description of pyramids; examples of pyramids in the real world

These topics are optional.

CHART 12

Modified Mathematics — Course C
Level 1, Grade 11; Level 2, Grade 10

C Mod

Numerical Methods (N)

Section 1. Non-computational Use of Numbers
a) Unrestricted counting by 2, 3, 4, 5, 10, 20, 25; notion of finite sequences, infinite sequences
b) Writing numbers (unrestricted) using numerals, words
c) Reading numerals
d) Arranging numbers in order; using numbers to sequence things
e) Ranking things as first, second, third, etc.
f) Simple codes; identifying parts of a code; coding and decoding

Section 2. Computational Use of Whole Numbers
a) Place value
b) Adding and subtracting numbers with up to four digits, in columns and in rows
c) Multiplying one- to four-digit numbers; squaring numbers 1, 2, 3, . . . , 10; cubing numbers 1, 2, 3, 4
d) Factoring one- and two-digit numbers; perfect squares and their square roots
e) Dividing by one- to three-digit numbers
f) Properties of addition and multiplication, developed informally
g) Reasonableness of answers
h) Approximating
i) Estimating

Section 3. Computational Use of Decimals
a) Place value (unrestricted)
b) Adding and subtracting decimals in columns and in rows
c) Multiplying decimals
d) Dividing decimals by one- to three-digit numbers
e) Multiplying and dividing a decimal by 10, 100, 1000, etc., and by 0.1, 0.01, etc.
f) Reasonableness of answers
g) Approximating
h) Estimating
i) Rounding and truncating

Section 4. Percent
a) Meaning of percent
b) Finding a percent of a number; applications in real-life situations
c) Finding the percent of a part to the whole; applications in real-life situations

Section 5. Problem Solving
a) Solving a variety of real-world and mathematical problems, by a method such as:
- identifying the problem
- collecting and organizing the significant information
- analysing this information and developing a strategy
- executing the strategy and testing the reasonableness of the result
- presenting the solution if the result is reasonable, or repeating the above

Section 6. Fractions*
a) Fraction as part of a whole, as part of a group
b) Equivalent fractions; reduction of fractions (simple cases only)
c) Adding and subtracting fractions with the same denominators, with different denominators
d) Multiplying a whole number by a fraction, a fraction by a fraction
e) Decimal equivalents of fractions, such as $\frac{1}{2}$, $\frac{2}{5}$, $\frac{3}{4}$, and $\frac{1}{3}$

* with denominators of 2, 4, 8, 3, 5, 10

Section 7. Data Graphs
a) Reading, comparing, and interpreting information from charts, graphs, and tables
b) Collecting and organizing data from real-life situations, from experiments; polling, surveying, sampling
c) Representing data by pictographs, bar graphs, circle graphs, and line graphs
d) Representing data in tables
e) Mode, median, and mean of the data
f) Graphing on a coordinate grid

Section 8. Measurement*
a) Measuring length, estimating
b) Measuring area, estimating
c) Measuring volume, estimating
d) Measuring capacity, estimating
e) Measuring mass, estimating
f) Measuring temperature, estimating
g) Measuring pressure
h) Selecting the appropriate unit; equivalent units and conversion

* using metric units cm, m, km, cm^2, m^2, ha, cm^3, m^3, mL, L, g, kg, t, °C, kPa, and imperial units until they are no longer in common use

Section 9. Probability
a) Counting the outcomes of experiments
b) Formula for simple probability
c) Finding the probability using a tree diagram
d) The Fundamental Counting Principle; applications
e) Odds
f) Dependent events; independent events

Geometry (G)

Section 1. Motion Geometry
a) Drawing a slide image on a grid, on plain paper; identifying slide images in diagrams and in the real world
b) Drawing a flip image on a grid, on plain paper; identifying flip images in diagrams and in the real world; symmetry
c) Drawing a turn image on a grid, on plain paper; identifying turn images in diagrams and in the real world; symmetry
d) Drawing an enlarged or reduced image on a grid; size and shape; identifying enlarged or reduced images in diagrams and in the real world; scale drawings
e) Drawing a distortion image using a distorted grid

Section 2. Geometric Constructions
a) Using a protractor to measure and construct angles from 0° to 180°; estimating the size of angles
b) Constructing circles using a compass; centre, radius, diameter
c) Constructing triangles using a variety of techniques; right-angled, isosceles, equilateral, scalene triangles
d) Constructing squares and rectangles using a variety of techniques
e) Using a compass and protractor to construct a square, equilateral triangle, and other regular polygons in a circle; symmetry
f) Tiling patterns; tiling with triangles, quadrilaterals, regular hexagons; geometric properties; mosaics; symmetry

Section 3. Prisms and Pyramids
a) Identifying prisms by the base; vertex, edge, face; characteristics of prisms; examples of prisms in the real world; right prisms and symmetry
b) Identifying pyramids by the base; vertex, edge, face; characteristics of pyramids; examples of pyramids in the real world; right pyramids and symmetry
c) Constructing prisms and pyramids as shells, as skeletons
d) Stacking space

These topics are optional.

Planning the School's Program

Introduction

The mathematics program of a school consists of the mathematics courses that it provides for its students. The courses for Grades 7 and 8, called Grade 7 Mathematics and Grade 8 Mathematics, are based on the outlines of topics for these grades. They may be offered at different levels of difficulty to meet the needs of special groups within the school. Some suggestions for adapting these courses to specific situations are provided below.

The courses for Grades 9 and 10 are to be based on the outlines of topics provided for the four levels of difficulty for these grades. Each course is named by its grade and level of difficulty (see the Preface). A school need not necessarily offer courses at each level of difficulty; local conditions will determine which combination of courses is appropriate and feasible for the school's student body. Considerable flexibility is built into each course to allow modifications dictated by the needs of students. This aspect of course development is discussed below.

There are many factors that will have a bearing on the nature of the program developed in the school. These include:

- the principles and directions set out in this mathematics guideline;
- the influence of local curriculum committees;
- the background of teachers;
- the background of the students (ability, interests, needs);
- the availability of human resources (other staff members; mathematics co-ordinator; mathematics consultants; school librarian; staff of faculties of education, of universities and community colleges, of the Ontario Institute for Studies in Education; members of the Ontario Association of Mathematics Educators; other people in the community with special expertise);
- the character and needs of the community;
- the availability of physical resources (Ministry and locally developed resource notes, textbooks, resource books, resource journals, calculators, computers, TV, films, film loops, overhead projectors, supplies, photocopier, etc.);
- in-service and professional development activities;
- school facilities;
- programs in other subjects and out-of-school education;
- specific features of local school policy (class organization, timetabling, the time provided for classroom instruction, the physical nature of the classroom, evaluation policies, reporting procedures, promotion policy, etc.).

Flexibility in Local Curriculum Planning

The mathematics program outlined in this guideline is intended to provide students with opportunities to learn facts, concepts, and skills that are or will be essential in their lives. This expectation has vastly different program implications for different students. On the one hand, the school's program must encompass the requirements of future courses in mathematics and of other subjects both in the current year and in the future. In some instances this involves providing students with knowledge that is immediately transferable; in other instances it involves providing the foundations upon which specific skills, applications, and concepts can be built in concurrent and subsequent courses. On the other hand, the program must provide the knowledge and applications that are essential for living in today's world – for functioning in the world of business and industry, in the market place, and in the home; for participation in recreational activities; for financial planning; for understanding the environment; for travel; and so on.

These needs are not uniform; they vary from person to person. Furthermore, many of them cannot be clearly identified now for all students or for all possible situations that might occur in a world that is constantly changing. Thus it is essential that the school's curriculum be flexible and adaptable.

Each of the courses outlined in this guideline is designed to facilitate the local development of courses that will accommodate the diverse needs, interests, and aptitudes of a wide range of students. Although each course must be based on the core program set out in its outline of topics, a large measure of individualization can be achieved through the following:

- the choice of optional topics selected for study;
- the depth and breadth of study accorded core and optional topics;
- the methodologies used in the classroom development of the course;
- the mathematical approaches used in the classroom development of the course;

- the emphasis given to the review and consolidation of fundamental ideas from earlier studies;
- the sequencing of topics within the course;
- the extent to which topics are interrelated and treated in a unified way in the development of the course;
- the emphasis given to applications and problem-solving;
- the nature and scope of the evaluation processes used, as well as the expectations set for students.

Since, under normal circumstances, the core topics should require only 75 percent of the instructional time, with the balance of the course based on appropriate optional topics, a significant measure of flexibility exists in the choice of topics. However, in some instances students within a class may require additional emphasis on the core topics – either through remediation related to basic concepts and skills, or through a fuller development of some of the core topics than is implied by the time frames suggested for the core topics in this guideline.

It is suggested that in planning their course teachers initially follow the time pattern provided in this guideline. Modifications can then be made to accommodate student interests and needs. Since both core and optional topics are identified here in general terms, each teacher can establish specific objectives for each topic in keeping with the needs and interests of his or her students. In this way, all schools in the province will include the same core topics in their courses, but the scope of the development of these topics and of the selected optional topics will vary from class to class.

It is within the policy of this guideline for a school or a school system to plan a core program that goes beyond the provincial core program identified here, while including all aspects of it. In such cases, it is expected that local school authorities will ensure that the core program adopted is in the best interests of the students concerned.

The Scope of Courses

The scope of a course can be varied through the selection of topics, the order and timing of topics, the breadth and depth of the study, the mathematical approaches used, the methodologies adopted, the applications investigated, and the evaluation procedures used.

Core and optional topics
All of the core topics set out in the outline of topics for a course must be included in the locally planned course, and all students should be familiar with them by the time they complete the course. These topics are fundamental to the development of future courses at the same level of difficulty and in some instances to other subjects in the curriculum. Optional topics are combined with the core topics to form the program content of each course. It is anticipated that, in the years ahead, as the core program becomes more firmly established in earlier grades, less time will be needed to develop the core program and more time will be available for the inclusion of optional topics.

Order of topics
The organization of the outlines of topics into strands, sections, and topics establishes related clusters of ideas. Since many topics are interrelated with other topics within the same strand and in other strands, there are many opportunities within each course for introducing, developing, reviewing, and applying the same topics. Such a unified development will help establish the significance and relevance of different ideas within each course. It is not intended that the order of topics outlined in this guideline be construed as the optimal or only order to be followed.

Time plan
The time suggestions provided in this guideline were worked out by the Validation Committee and are provided as an aid to teachers in planning their courses. These suggestions need not be strictly followed. It is suggested, however, that teachers use the time designations as a guide in deciding on the emphases to be adopted in the development of different sections of their courses.

Breadth and depth of treatment
A topic may have a variety of objectives. The breadth of study established for a topic should be based not only on the number of these objectives, but also on the number and variety of situations that are to be investigated for each objective.

The depth of study for the topic involves the difficulty of the problem situations associated with the topic's objectives, the precision that is expected in oral and written work, and the emphasis that is to be placed on rigour and structure.

Mathematical approaches
The approach to a topic can vary from an informal, intuitive development to a formal approach that emphasizes rigour, structure, and proof. It is strongly suggested that topics be introduced through informal approaches and that, even in the advanced level program, students not be expected to use formal approaches until they give some indication of the maturity required. On this basis, it is recommended that attempts to use formal approaches be delayed until late in the Intermediate Division and then be used essentially in the Advanced Program and only with some students for some topics.

A further refinement involves the mathematical vehicle, or combination of vehicles, used to develop the topic. For example, many properties of a parallelogram can be developed informally by using measurement or tracing paper and physical motions, or, more formally, by using a transformation approach, a traditional Euclidean geometry approach, a vector approach, or coordinate geometry.

Methodologies

Many methodologies can be used effectively within the classroom. In view of the ongoing need for education in our changing world, the methods that elicit independent thought and student involvement in the development of the lesson and that encourage independent study habits are in the best interests of the students in the long run. A variety of methods should be used in order to provide a change of pace in the classroom and to give students opportunities to learn to adapt to different learning situations. The following are some of the methods by which the program can be developed: Socratic lessons; group investigations and discussion; individual student assignments; lessons using audio-visual aids (e.g., overhead projector, TV, films, film loops); the construction and use of models; laboratory situations; student-led seminars; the use of mathematical games and puzzles; lectures; the use of out-of-school resource people; field trips; lessons based on problem solving; lessons using calculators or computers; the use of worksheets; programmed learning units; diagnostic remedial lessons; small-group work with student leaders; and essays and other projects.

Applications

Applications can be used both to establish a need for a new concept or skill and to consolidate skills that have already been established. They help students to recognize the relevance of the topics studied and to apply a number of these topics in solving problems.

Evaluation

Purpose

Evaluation of student achievement and program effectiveness must be an integral, ongoing part of the teaching-learning process. It can serve the following functions:

- to provide the student with information concerning what he or she has already mastered and what still needs to be learned;
- to provide parents with information regarding their child's progress;
- to verify the appropriateness of stated goals and objectives;
- to provide information that will enable teachers to modify the program as needed;
- to assess the quality and effectiveness of the learning strategies and materials that have been used;
- to spur the student to further achievement.

Sources of Information

Because evaluation is concerned with obtaining information needed to assess various aspects of the teaching-learning process, it is necessary to use information obtained from a variety of sources. These might include the following:

- records of previous achievement
- topic or unit pre-tests
- systematic observation of attitudes and performance
- oral and written presentations of solutions to problems and of mathematical applications
- personal interviews
- projects
- quizzes related to specific objectives
- topic or unit post-tests
- examinations at specific intervals
- suitable standardized tests

Strategies

As effective learning is dependent on the learner's readiness and background, teachers should attempt, as a first priority, to establish a profile of individual attitudes and competencies. With this information at hand, an individualized sequence of learning objectives can be established for each student. As learning progresses, the student must be provided with immediate and continuing feedback so that he or she may be able to monitor his or her own progress. A more general assessment of the success with which broader outcomes have been attained throughout some significant part of the course is also an important component of evaluation.

Initial evaluation processes that are directed towards identifying characteristics of the learner depend on both static and dynamic sources of information. Individual records indicate performance levels at particular points in the learner's past development. These should be brought up to date through systematic observation of student-teacher interaction and careful interpretation of test results. For this information to be most useful, a systematic record system is absolutely vital.

Studies have shown that the practice of keeping "field notes" is invaluable. On the one hand, it helps teachers to focus on individuals and, as a result, to become aware of aspects of attitude and behaviour that were formerly unnoticed; on the other, it can motivate students by helping them to see the validity of the evaluation and to act on the information revealed. As well, it provides a sound context within which to interpret test and examination results and other data. The establishment of such a context is vital in assessing student progress: for the student, for his or her parents, and for the school.

The Ministry of Education is currently developing materials in a number of subject areas that will assist teachers, principals, and school board officials in monitoring programs and assessing student achievement. These materials will be part of the Ontario Assessment Instrument Pool. The first round of distribution of instruments for Intermediate Division Mathematics will take place in the fall of 1980, and will contain instruments designed to assist teachers in developing strategies for student assessment as well as test items coded to the content areas of the guideline. The characteristics of each item will be described so that teachers may select the items best suited to a particular purpose such as the development of a survey test, an achievement test, or a diagnostic test.

Students seldom learn only one thing at a time. Evaluation of achievement in mathematics has been hindered by the misconception that the learning process is straightforward, sequential, and, hence,

easy to measure. This is not the case. Assessment practices must take into account the complexity of learning. They must deal with specific learning tasks, with degrees of understanding, and with values and attitudes, as well as with facts, skills, and concepts. Written tests or examinations, no matter how well they are constructed, do not provide all the information needed for a valid assessment. It is only after data from interviews, teacher observation, projects, and tests have been sorted and studied that a sound professional judgement can be made.

Most of the teacher's assessments will be formative, aimed at providing individuals or groups of students with feedback concerning their progress over the short term. The teacher can then decide whether or not the learning situation has been effective and can provide for changes in pacing, in materials, in teaching methods, or in objectives. An analysis of the effectiveness of all aspects of the program, as well as reports on the progress made by students, should be made at regular intervals.

The practice of using evaluation, examinations, and the threat of failure to motivate students can sometimes be counterproductive. In fact, the anxiety generated by such strategies often prevents students from doing their best, and the results are not indicative of either the level of their achievement or the quality of the program. By generating genuine interest in the subject and by providing a reasonable and systematic process of evaluation, the teacher can create a learning environment in which each student is encouraged to strive for the full realization of his or her potential.

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